EBA2911 Mathematics for Business Analytics autumn 2019
Exercises

I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.

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Lecture 14
Sec. 7.12, 6.4, 8.3, 8.5, 7.7
l'Hôpital's rule. Marginal revenue and cost. Elasticity.

Here are recommended exercises from the textbook [SHSC].

Section **7.12** exercise 1-3, 4a, 5
Section **6.4** exercise 2, 6
Section **8.5** exercise 1-4
Section **7.7** exercise 1-3

Problems for the exercise session
Wednesday 13 Nov. from 14 o'clock in B2-085

Problem 1 Compute the limit values.

a) $\displaystyle\lim_{x\to 3}\frac{-x}{25(x-1)}$ b) $\displaystyle\lim_{x\to \ln 5}\frac{e^x-5}{x^2-5}$ c) $\displaystyle\lim_{x\to \ln 5}\frac{e^x-5}{x^2-(\ln 5)^2}$

d) $\displaystyle\lim_{x\to 0}\frac{7x}{e^x-1}$ e) $\displaystyle\lim_{x\to 0}\frac{x^{10}}{e^x-1}$ f) $\displaystyle\lim_{x\to 1}\frac{x\ln(x)}{x^2-1}$

g) $\displaystyle\lim_{x\to 1}\frac{\ln(x)}{e^{2x}-e^2}$ h) $\displaystyle\lim_{x\to 1}\frac{\ln(x)}{\sqrt{x}-1}$ i) $\displaystyle\lim_{x\to 2}\frac{e^{x^2-3x+2}-1}{x^2-4}$

Problem 2 Compute the limit values by applying l'Hôpital's rule.

(a) $\displaystyle\lim_{x\to \infty}\frac{-x}{25(x-1)}$ (b) $\displaystyle\lim_{x\to \infty}\frac{x^2-4x+10}{e^x-5}$ (c) $\displaystyle\lim_{x\to \infty}\frac{\ln(x)}{x}$

Problem 3 Explain why $C(x)$ is a cost function by checking the three criteria:
(1) $C(0) > 0$
(2) $C(x)$ is an increasing function
(3) $C(x)$ is a convex function
Also determine the cost optimum and the average cost per unit at cost optimum (also called *the minimal unit cost* or *the optimal unit cost*).

a) $C(x) = 0.01x^2 + 8x + 2500,\ x \geqslant 0$ b) $C(x) = 0.05(x+200)^2,\ x \geqslant 0$

c) $C(x) = 400e^{0.001x^2},\ x \geqslant 0$ d) $C(x) = 1000e^{0.0004(x+5)^2},\ x \geqslant 0$

e) $C(x) = 50x + 1000,\ 0 \leqslant x \leqslant 1000$

Problem 4 $C(x)$ is the cost function, $R(x)$ is the revenue function and x is number of produced and sold units. Determine the profit maximising number of units.
a) $C(x) = 0.01x^2 + 8x + 2500$ and $I(x) = 100x$ for $x \geqslant 0$
b) $C(x) = 0.005x^2 + 20x + 30000$ and $I(x) = 50x$ for $0 \leqslant x \leqslant 2000$

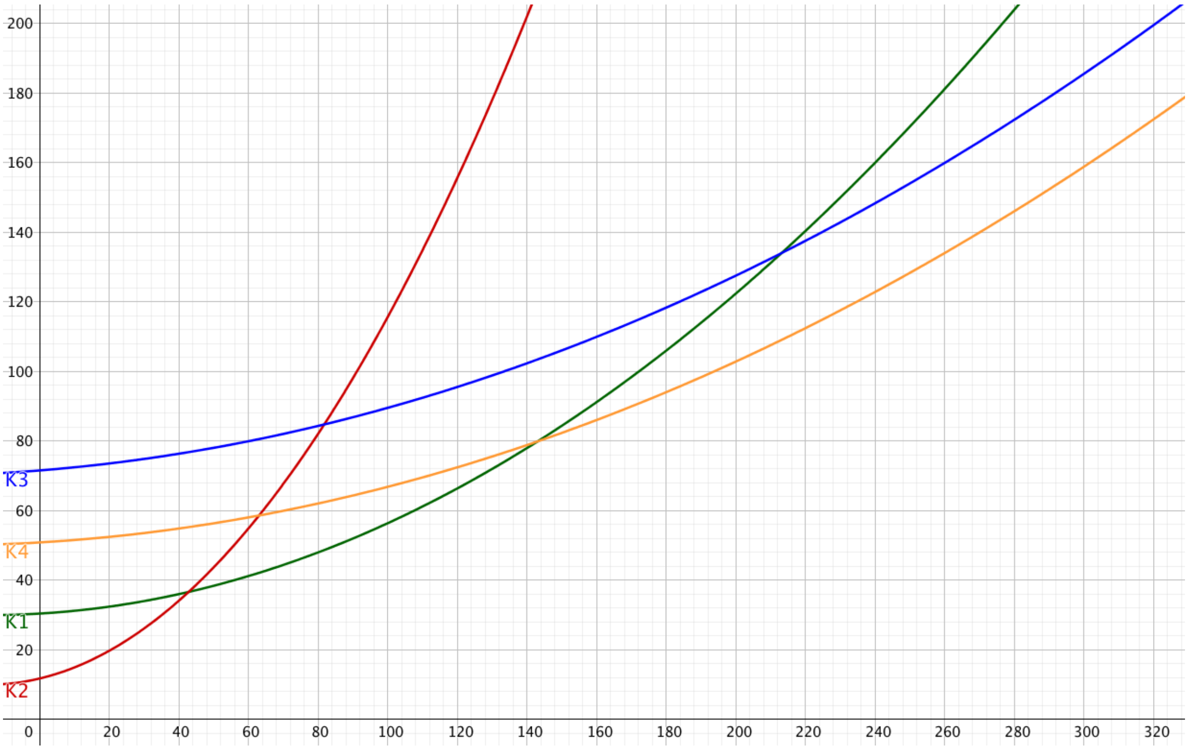

Figure 1: Fire kostnadsfunksjoner (K_1–K_4)

Problem 5 I figure 1 you see the graph of four different cost functions.

a) Order the cost functions from the one with the smallest minimal unit cost to the one with with the largest minimal unit cost.
b) Find an approximate value for the cost optimum for each of the cost functions.
c) Find an approximate value for the minimal unit cost for each of the cost functions.

Problem 6 Let p be the price of a commodity and $D(p)$ the demand for the commodity with price p (so $D(p)$ is the number of sold units). Determine the relative change of price, the relative change of demand and the (average) elasticity of the demand with respect to price. Determine if the demand is elastic, inelastic, or unit elastic.

a) $D(30) = 40$ and $D(30.5) = 39$
b) $D(20) = 101$ and $D(21) = 100.95$
c) $D(10) = 24.648$ and $D(10.01) = 24.623$

Problem 7 Let p be the price of a commodity and $D(p)$ the demand for the commodity with price p (so $D(p)$ is the number of sold units). Determine the (momentary) elasticity $\varepsilon(p) = \mathrm{El}_p(D(p))$ of the demand with respect to price. Determine the price p such that the demand is elastic, inelastic, and unit elastic.

a) $D(p) = 100 - 2p$ med $0 < p < 50$
b) $D(p) = 100 + \frac{20}{p}$ med $p \geqslant 1$
c) $D(p) = 67e^{-0.1p}$ med $p > 0$
d) $D(p) = 100 + \frac{900}{p^2}$ med $p \geqslant 1$
e) $D(p) = 53e^{-0.02p^2}$ med $p > 0$

Answers

Problem 1

a) $\frac{-3}{25(3-1)} = -0.06$	b) 0	c) $\frac{5}{2\ln 5}$
d) 7	e) 0	f) 0.5
g) $\frac{1}{2e^2}$	h) 2	i) $\frac{1}{4}$

Problem 2

a) $\frac{-1}{25}$	b) $\lim_{x\to\infty} \dfrac{2}{e^x} = 0$	c) 0

Problem 3

a) $C(0) = 2500 > 0$, $C'(x) = 0.02x + 8 > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0.02 > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 500$ gives minimal unit cost $A(500) = 18$

b) $C(0) = 2000 > 0$, $C'(x) = 0.1x + 20 > 0$ for $x > 0$ and so $C(x)$ is a increasing function for $x \geqslant 0$, $C''(x) = 0.1 > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 200$ gives minimal unit cost $A(200) = 40$

c) $C(0) = 400 > 0$, $C'(x) = 0.8xe^{0.001x^2} > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0.8(1 + 0.002x^2)e^{0.001x^2} > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 22.36$ gives minimal unit cost $A(22.36) = 29.49$

d) $C(0) = 1010.05 > 0$, $C'(x) = 0.8(x+5)e^{0.0004(x+5)^2} > 0$ for $x > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0.8[1 + 0.0008(x+5)^2]e^{0.0004(x+5)^2} > 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 32.94$ gives minimal unit cost $A(32.94) = 53.99$

e) $C(0) = 1000 > 0$, $C'(x) = 50 > 0$ and so $C(x)$ is an increasing function for $x \geqslant 0$, $C''(x) = 0 \geqslant 0$ and so $C(x)$ is a convex function for $x \geqslant 0$. Cost optimum $x = 1000$ gives minimal unit cost $A(1000) = 51$

Problem 4

a) For $x = 4600$ the marginal cost equals the marginal revenue and $\pi''(x) = -0.02 < 0$ gives that the profit function is concave and hence $x = 4600$ is maximising the profit.

b) For $x = 3000$ the marginal cost equals the marginal revenue, but this is outside the domain of definition for the modell. We see that $\pi'(x) = 30 - 0.01x$ is positive for $x < 3000$ which gives that the profit function is increasing for x in the interval $[0, 2000]$ and hence $x = 2000$ is maximising the profit.

Problem 5

a) K_4, K_1, K_3, K_2

b) $K_4 : \ x = 220, \ K_1 : \ x = 120, \ K_3 : \ x = 270, \ K_2 : 40$

c) $A_4(220) = \frac{112}{220} = 0.51, \ A_1(120) = \frac{65}{120} = 0.54, \ A_3(270) = \frac{165}{270} = 0.61, \ A_2(40) = \frac{35}{40} = 0.88$

Problem 6

a) the relative change of price is $\frac{0.5}{30}$, the relative change of demand is $\frac{-1}{40}$ and the elasticity is -1.5, i.e. elastic demand

b) the relative change of price is $\frac{1}{20}$, the relative change of demand is $\frac{-0.05}{101}$ and the elasticity is -0.0099, i.e. inelastic demand

c) the relative change of price is 0.001, the relative change of demand is -0.001014 and the elasticity is -1.014, i.e. elastic demand

Problem 7

a) $\varepsilon(p) = \frac{-2p}{100-2p}$. The demand function is unit elastic for $p = 25$, inelastic for $0 < p < 25$ and elastic for $25 < p < 50$.
b) $\varepsilon(p) = -\frac{1}{5p+1}$. The demand function is inelastic for all $p \geqslant 1$.
c) $\varepsilon(p) = -0.1p$. The demand function is unit elastic for $p = 10$, inelastic for $0 < p < 10$ and elastic for $p > 10$.
d) $\varepsilon(p) = -\frac{18}{p^2+9}$. The demand function is unit elastic for $p = 3$, elastic for $1 \leqslant p < 3$ and inelastic for $p > 3$.
e) $\varepsilon(p) = -0.04p^2$. The demand function is unit elastic for $p = 5$, inelastic for $0 < p < 5$ and elastic for $p > 5$.