## Key Problems

## Problem 1.

Solve the systems of equations:
a) $2 x+3 y=14$
b) $x^{2}+y^{2}=20$
c) $x-2 y=6$
$x y=-4$
d) $x^{2}-y^{2}=8$
$7 x-4 y=20$
$x-y=2$
$x y=3$

## Problem 2.

Solve the equation $a x=b$ when
a) $a=b=1$
b) $a=1, b=0$
c) $a=0, b=1$
d) $a=b=0$

## Problem 3.

Solve the systems of equations:
a) $x+y+z=4$
$x+2 y+4 z=9$
$x+3 y+9 z=16$
b) $\quad x-y+z=3$
$2 x-4 y+z=1$
$3 x-5 y+2 z=4$

## Problem 4.

Use Gaussian elimination to solve the linear systems:
$x+y+z=11$
$x+y+z=6$
a) $x+2 y+4 z=22$
$x-y+z=1$
b) $x+2 y+4 z=16$
$x+3 y+9 z=20$

## Problem 5.

Use Gaussian elimination to solve the linear systems. How many solutions are there?
a) $\begin{aligned} x+3 y & =1 \\ x-y & =9 \\ 2 x+2 y & =3\end{aligned}$
b) $\begin{aligned} x+3 y & =7 \\ x-y & =3 \\ 2 x+2 y & =10\end{aligned}$
$\begin{aligned} x+y+z & =11 \\ x-y+z & =9 \\ x+3 y+5 z & =44 \\ 2 x+3 y & =45\end{aligned}$

## Problem 6.

Use Gaussian elimination to solve the linear systems. How many solutions are there?
а) $\begin{aligned} x+2 y+3 z & =4 \\ -x-y+z & =1 \\ 3 x+4 y+z & =2\end{aligned}$
b) $\begin{aligned} 3 x+4 y+3 z & =2 \\ 2 x-y+z & =1 \\ 7 x+2 y+5 z & =3\end{aligned}$

## Problem 7.

Use Gaussian elimination to solve the linear system. How many solutions are there?

$$
\left.\begin{array}{rl}
x+y+z & + \\
x+2 y+ & + \\
x & + \\
x & -y \\
x & z
\end{array}\right)
$$

## Problem 8.

A linear system is called homogeneous if all constant terms are zero. How many solutions does a homogeneous linear system with three equations and five variables have?

## Problem 9.

Solve the system of equations:

$$
\begin{array}{r}
2 x y+y^{3}+y^{2}=0 \\
x^{2}+3 x y^{2}+2 x y=0
\end{array}
$$

## Problem 10.

Optional: Problems from [Eriksen] (norwegian textbook)
Problem 6.1.1-6.1.6, 6.2.1-6.2.5, 6.3.1-6.3.7 (textbook)

## Answers to Key Problems

## Problem 1.

a) $(x, y)=(4,2)$
b) $(x, y)=(4,2),(-2,-4)$
c) $(x, y)=(2,-2),(4,-1)$
d) $(x, y=(3,1),(-3,-1)$

## Problem 2.

a) $x=1$
b) $x=0$
c) no solutions
d) all values of $x$ are solutions

## Problem 3.

a) $(x, y, z)=(1,2,1)$
b) $(x, y, z)=(-z / 2+1 / 2, z / 2-5 / 2, z)$ where $z$ is a free variable

## Problem 4.

a) $(x, y, z)=(4,5,2)$
b) $(x, y, z)=(-10,19,-3)$

## Problem 5.

a) No solutions
b) One solution $(x, y)=(4,1)$
c) No solutions

## Problem 6.

a) Infinitely man solutions $(x, y, z)=(-6+5 z, 5-4 z, z)$ with $z$ free
b) No solutions

## Problem 7.

Infinitely man solutions $(x, y, z)=(13-5 w,-3+5 w,-w, w)$ with $w$ free

## Problem 8.

Infinitely many solutions.

## Problem 9.

Solutions: $(x, y)=(0,0),(0,-1),(3 / 25,-3 / 5)$

