# Key Problems

## Problem 1.

Solve the systems of equations:

a) 
$$2x + 3y = 14$$
  
 $7x - 4y = 20$   
b)  $x^2 + y^2 = 20$   
 $x - y = 2$   
c)  $x - 2y = 6$   
 $xy = -4$   
d)  $x^2 - y^2 = 8$   
 $xy = 3$ 

## Problem 2.

Solve the equation ax = b when

a) a = b = 1 b) a = 1, b = 0 c) a = 0, b = 1 d) a = b = 0

## Problem 3.

Solve the systems of equations:

a) 
$$x + y + z = 4$$
  
 $x + 2y + 4z = 9$   
 $x + 3y + 9z = 16$ 
b)  $x - y + z = 3$   
 $2x - 4y + z = 1$   
 $3x - 5y + 2z = 4$ 

#### Problem 4.

Use Gaussian elimination to solve the linear systems:

|    | x | + | y  | + | z  | = | 11 |    | x | + | y  | + | z  | = | 6  |
|----|---|---|----|---|----|---|----|----|---|---|----|---|----|---|----|
| a) | x | + | 2y | + | 4z | = | 22 | b) | x | + | 2y | + | 4z | = | 16 |
|    | x | _ | y  | + | z  | = | 1  |    | x | + | 3y | + | 9z | = | 20 |

## Problem 5.

Use Gaussian elimination to solve the linear systems. How many solutions are there?

|    |    |   | 9  |   | 1 |            |    |   | 9  |   | -  |        | x                 | + | y              | + | z              | = | 11 |
|----|----|---|----|---|---|------------|----|---|----|---|----|--------|-------------------|---|----------------|---|----------------|---|----|
| `  | x  | + | 3y | = | 1 | - \        | x  | + | 3y | = | (  | 、<br>、 | x                 | _ | $\overline{u}$ | + | z              | = | 9  |
| a) | x  | _ | y  | = | 9 | <i>b</i> ) | x  | _ | y  | = | 3  | c)     | 2x                | + | 3u             | + | 5z             | _ | 44 |
|    | 2x | + | 2y | = | 3 |            | 2x | + | 2y | = | 10 |        | $\frac{2\pi}{3x}$ | _ | 09<br>11       | + | $\frac{3}{2z}$ | _ | 45 |

## Problem 6.

Use Gaussian elimination to solve the linear systems. How many solutions are there?

|    | x  | + | 2y | + | 3z | = | 4 |    | 3x | + | 4y | + | 3z | = | 2 |
|----|----|---|----|---|----|---|---|----|----|---|----|---|----|---|---|
| a) | -x | — | y  | + | z  | = | 1 | b) | 2x | — | y  | + | z  | = | 1 |
|    | 3x | + | 4y | + | z  | = | 2 |    | 7x | + | 2y | + | 5z | = | 3 |

## Problem 7.

Use Gaussian elimination to solve the linear system. How many solutions are there?

| x | + | y  | + | z  | + | w   | = | 10             |
|---|---|----|---|----|---|-----|---|----------------|
| x | + | 2y | + | 4z | — | w   | = | $\overline{7}$ |
| x | _ | y  | + | z  | + | 11w | = | 16             |

#### Problem 8.

A linear system is called *homogeneous* if all constant terms are zero. How many solutions does a homogeneous linear system with three equations and five variables have?

#### Problem 9.

Solve the system of equations:

$$2xy + y3 + y2 = 0$$
$$x2 + 3xy2 + 2xy = 0$$

## Problem 10.

Optional: Problems from [Eriksen] (norwegian textbook) Problem 6.1.1 - 6.1.6, 6.2.1 - 6.2.5, 6.3.1 - 6.3.7 (textbook)

## Answers to Key Problems

#### Problem 1.

a) (x,y) = (4,2)b) (x,y) = (4,2), (-2, -4)c) (x,y) = (2, -2), (4, -1)d) (x,y = (3,1), (-3, -1)

## Problem 2.

a) x = 1

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*v* — 1

b) x = 0

c) no solutions

d) all values of x are solutions

#### Problem 3.

a) (x,y,z) = (1,2,1) b) (x,y,z) = (-z/2 + 1/2, z/2 - 5/2, z) where z is a free variable

#### Problem 4.

a) (x,y,z) = (4,5,2) b) (x,y,z) = (-10,19,-3)

#### Problem 5.

a) No solutions b) One solution (x,y) = (4,1) c) No solutions

## Problem 6.

a) Infinitely man solutions (x,y,z) = (-6 + 5z, 5 - 4z, z) with z free b) No solutions

#### Problem 7.

Infinitely man solutions (x,y,z) = (13 - 5w, -3 + 5w, -w, w) with w free

**Problem 8.** Infinitely many solutions.

#### Problem 9.

Solutions: (x,y) = (0,0), (0, -1), (3/25, -3/5)