Key Problems

Problem 1.
Compute the lengths of the vectors $\mathbf{u}$ and $\mathbf{v}$, and the area of the parallelogram spanned by these vectors. Show the area in a figure.

a) $\mathbf{u} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ b) $\mathbf{u} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ c) $\mathbf{u} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$

Problem 2.
Compute the lengths of the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$, and the volume of the parallelepiped spanned by these vectors. Show the volume in a figure, if you can.

a) $\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ b) $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

Problem 3.
Compute the determinants:

a) $\begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix}$ b) $\begin{vmatrix} 7 & 3 \\ 3 & 1 \end{vmatrix}$ c) $\begin{vmatrix} 3 & 1 \\ 7 & 3 \end{vmatrix}$ d) $\begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix}$ e) $\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$ f) $\begin{vmatrix} a & b \\ b & c \end{vmatrix}$

Problem 4.
Compute the determinants:

a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{vmatrix}$ c) $\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & 5 \end{vmatrix}$ d) $\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{vmatrix}$ e) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix}$

Problem 5.
Compute the determinants, and determine when the determinants are zero:

a) $\begin{vmatrix} 2 & a \\ a & 8 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & a & 9 \end{vmatrix}$ c) $\begin{vmatrix} a & 1 & 7 \\ 0 & 1-a & a \\ 0 & 0 & 2a \end{vmatrix}$ d) $\begin{vmatrix} 1 & 2 & a \\ 1 & a & 3 \\ 1 & a & 1 \end{vmatrix}$ e) $\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix}$

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Problem 6.
Compute the determinants:

a) $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix}$ b) $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 2 \end{vmatrix}$ c) $\begin{vmatrix} 1 & 1 & 4 & 6 \\ 0 & 2 & \sqrt{3} & -1 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & -2 \end{vmatrix}$

Problem 7. *Python*
Write a Python script that defines a function Gauss(x,n), which returns an approximate value of the expression

$$\frac{1}{2} + \int_0^x \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \, dz$$

using a Riemann sum with n subintervals. This gives the cumulative distribution function $p(X \leq x)$ when X has standard normal distribution and $x \geq 0$. Then use the python script to compute

a) Gauss(1,10) b) Gauss(1,100) c) Gauss(1,1000) d) Gauss(1,10000)

Problem 8.
Optional: Problems from [Eriksen] (norwegian textbook)
Problem 6.4.1 - 6.4.3 (textbook)

Answers to Key Problems

Problem 1.
a) $||\mathbf{u}|| = ||\mathbf{v}|| = 5,\ A = 25$ b) $||\mathbf{u}|| = ||\mathbf{v}|| = 5,\ A = 24$ c) $||\mathbf{u}|| = ||\mathbf{v}|| = 13,\ A = 169$

Problem 2.
a) $||\mathbf{u}|| = ||\mathbf{w}|| = 1,\ ||\mathbf{v}|| = 2, \quad V = 2$ b) $||\mathbf{u}|| = ||\mathbf{w}|| = \sqrt{3},\ ||\mathbf{v}|| = \sqrt{21}, \quad V = 6$

Problem 3.
a) 2 b) -2 c) 2 d) -2 e) 0 f) $ac - b^2$

Problem 4.
a) 2 b) 2 c) 0 d) 6 e) $(1-a)(1-b)(b-a)$

Problem 5.
a) The determinant is $16 - a^2$, and it is zero when $a = \pm 4$
b) The determinant is $-a^2 + 2a + 7$, and it is zero when $a = 1 \pm \sqrt{8}$
c) The determinant is $2a^2(1-a)$, and it is zero when $a = 0$ or $a = 1$
d) The determinant is $4 - 2a$, and it is zero when $a = 2$
e) The determinant is $(a-1)^2(a+2)$, and it is zero when $a = 1$ or $a = -2$

Problem 6.
a) 4 b) -10 c) -12

Problem 7.
a) 0.84899 b) 0.84213 c) 0.84142 d) 0.84135