Key Problems

Problem 1.
Assume that A and B are 3×3-matrices with $|A| = 2$ and $|B| = -5$. Compute the determinants:

a) $\det(AB)$
b) $\det(3A)$
c) $\det(-2B^T)$
d) $\det(2A^{-1}B)$

Problem 2.
Let A be a 2×3-matrix.

a) Is A symmetric?
b) Is $A^T A$ symmetric?
c) Compute $A^T A$ when $A = \begin{pmatrix} 1 & -1 & 3 \\ 3 & 3 & 1 \end{pmatrix}$.

Problem 3.
Exam MET11803 12/2018
We consider the linear system $A \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} a & 1 & a \\ 1 & 2 & 3 \\ a & 3 & 0 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ -a \\ 3-a \end{pmatrix}$$

and a is a parameter.

a) **(6p)** Solve the linear system when $a = 1$.
b) **(6p)** Find the determinant $\det(A)$, and determine all values of a such that $\det(A) = 0$.
c) **(6p)** Determine all values of a such that $A \cdot \mathbf{x} = \mathbf{b}$ has infinitely many solutions.
d) **(6p)** Compute $A^2 - 3A$ when $a = 1$.

Problem 4.
Exam MET11803 06/2016
We consider the linear system $A \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 2-s & 3 & 3 \\ 3 & 2-s & 3 \\ 3 & 3 & 2-s \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{og} \quad \mathbf{b} = \begin{pmatrix} 3 \\ s+4 \\ 1-2s \end{pmatrix}$$

We consider s as a parameter and x,y,z as variables.

a) **(6p)** Solve the linear system when $s = 8$. How many degrees of freedom are there?
b) **(6p)** Compute $|A|$ for a general value of s.
c) **(6p)** Find A^{-1} when $s = 0$, and use A^{-1} to solve the linear system in this case.
d) **(6p)** Determine all values of s such that the linear system has exactly one solution, and find x in these cases.

Answers to Key Problems

Problem 1.

a) -10

b) 54

c) 40

d) -20

Problem 2.

a) No

b) Yes

c) $\begin{pmatrix} 10 & 8 & 6 \\ 8 & 10 & 0 \\ 6 & 0 & 10 \end{pmatrix}$

Problem 3.

a) $(x,y,z) = (2,0,-1)$

b) $|A| = -a(2a+3)$, and $|A| = 0$ for $a = 0$ and $a = -3/2$

c) $a = 0$

d) $\begin{pmatrix} 0 & 3 & 1 \\ 3 & 8 & -2 \\ 1 & -2 & 10 \end{pmatrix}$

Problem 4.

a) There is one degree of freedom for $s = 8$, and the solutions are given by $(x,y,z) = (z-2, z-3, z)$ where z is free.

b) $|A| = -s^3 + 6s^2 + 15s + 8$

c) $A^{-1} = \frac{1}{8}\begin{pmatrix} -5 & 3 & 3 \\ 3 & -5 & 3 \\ 3 & 3 & -5 \end{pmatrix}$ and $(x,y,z) = (0,-1,2)$ for $s = 0$.

d) For $s \neq -1, 8$, the system has exactly one solution with x-coordinate $x = 0$.