Key Problems

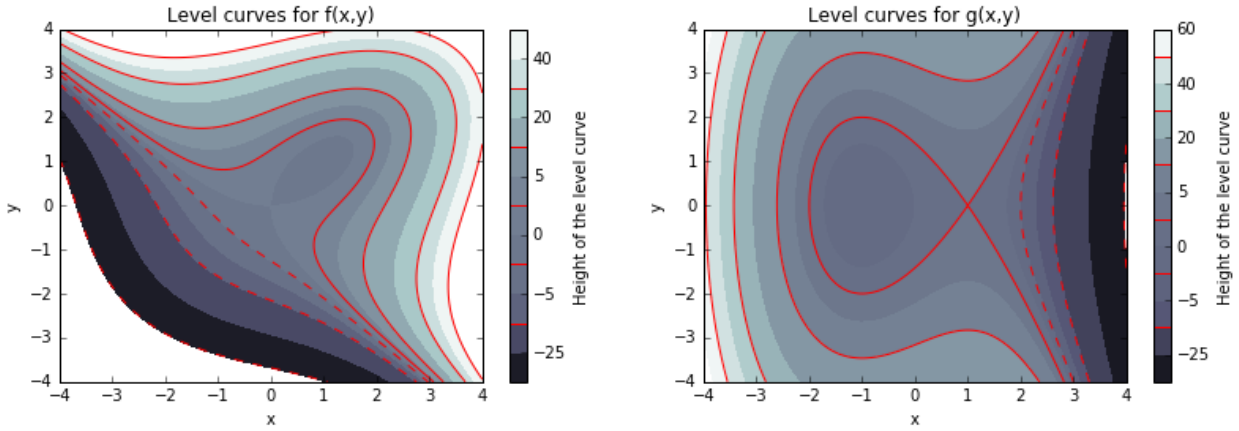
Problem 1.
Find the stationary points of f, and classify them:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 + y^2$
c) $f(x,y) = 4x^2 - 6xy + 9y^2$
d) $f(x,y) = x^2 - 2x + 4y^2$
e) $f(x,y) = x^3 - 3xy + y^3$
f) $f(x,y) = y^2 - x^3 + 3x$
g) $f(x,y) = \sqrt{x^2 + y^2}$
h) $f(x,y) = \ln(x^2y^2 - x^2 - y^2 + 3)$

Problem 2.
Find the linear approximation of f around the point $(x,y) = (1,1)$:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 + y^2$
c) $f(x,y) = 4x^2 - 6xy + 9y^2$
d) $f(x,y) = x^2 - 2x + 4y^2$
e) $f(x,y) = x^3 - 3xy + y^3$
f) $f(x,y) = y^2 - x^3 + 3x$

Problem 3.
Level curves for the functions f and g in the region $-4 \leq x,y \leq 4$ are shown in the figures below.



a) Find local maxima, minima and saddle points using the figures.
b) The functions f and g are two of the functions from Problem 1. Which?

Problem 4.
Find global maxima and minima, if they exist:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 + y^2$
c) $f(x,y) = 4x^2 - 6xy + 9y^2$
d) $f(x,y) = x^2 - 2x + 4y^2$
e) $f(x,y) = x^3 - 3xy + y^3$
f) $f(x,y) = y^2 - x^3 + 3x$
g) $f(x,y) = \sqrt{x^2 + y^2}$
h) $f(x,y) = \ln(x^2y^2 - x^2 - y^2 + 3)$

Problem 5.
Find the stationary points of f, and classify them:

a) $f(x,y) = xy(x^2 - y^2)$ b) $f(x,y) = x^2y + xy^3 + xy^2$ c) $f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$

Problem 6.
Problem 7.4.1 - 7.4.4, 7.5.1 - 7.5.5 (norwegian textbook, optional)

Answers to Key Problems

Problem 1.

a) none b) (0,0) is local min. c) (0,0) is local min. d) (1,0) is local min.

e) (0,0) is saddle point and (1,1) is local min. f) (1,0) is saddle point and $(-1,0)$ is local min.

g) none; (0,0) is critcal point h) (0,0) is local max and $(\pm 1, \pm 1)$ is saddle point

Problem 2.

a) $5 + 2(x-1) + 3(y-1)$ b) $2 + 2(x-1) + 2(y-1)$ c) $7 + 2(x-1) + 12(y-1)$

d) $3 + 8(y-1)$ e) -1 f) $3 + 2(y-1)$

Problem 3.

a) f has local min. (1,1) and saddle point (0,0), and g has local min. $(-1,0)$ and saddle point (1,0)

b) f is the function in e) and g is the function in f)

Problem 4.

a) no global max./min. b) (0,0) is global min. c) (0,0) is global min.

d) (1,0) is global min. e) no global max./min. f) no global max./min.

g) (0,0) is global min. h) no global max./min.

Problem 5.

a) (0,0) is saddle point b) (0,0), $(0,-1)$ are saddle points, $(3/25, -3/5)$ is local max.

c) (0,0) is local (and global) max.