EBA2911 Mathematics for Business Analytics autumn 2019
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 3
Sec. 10.4.1-3, 10.5.1-4
Total present value. Finite geometric series. Annuities, annuity mortage.

Here are recommended exercises from the textbook [SHSC].

Section **10.1** exercise 1-9
Section **10.3** exercise 1a, 2a
Section **10.4** exercise 1-3, 7
Section **10.5** exercise 1-5
Section **10.6** exercise 1, 2

Problems for the exercise session
Wednesday 28 Aug. at 14-16 in B2-085

Problem 1

(a) The ticket price Oslo-Bergen is changed from 699 to 899 NOK. Determine the relative change and the growth factor.

(b) The ticket price Oslo-Bergen is changed from 899 to 699 NOK. Determine the relative change and the growth factor.

(c) Determine the product of the growth factors in (a) and (b). Interpret the answer as a growth factor.

(d) Suppose a and b are two numbers different from 0. Show that

$$\left(1+\frac{b-a}{a}\right)\cdot\left(1+\frac{a-b}{b}\right)=1$$

and use this to explain the result in (c).

Problem 2 You deposit 50 000 in an account with 3.6% nominal annual interest.

(a) Suppose interest is added annually.
  (i) Calculate the balance after 10 years.
  (ii) Determine the growth factor and the relative change for these 10 years.

(b) Suppose interest is added monthly.
  (i) Calculate the balance after 10 years.
  (ii) Determine the growth factor and the relative change for these 10 years.
  (iii) Determine the effective annual interest.

Problem 3

(a) Determine what amount you have to deposit today to have 250 000 after 10 years if the interest is 3.4%.

(b) After 4 years the interest is changed to 1.9%. Determine the balance after 10 years.

(c) Explain why the answer in (b) is given by the expression $250\,000\cdot\left(\frac{1.019}{1.034}\right)^6$.

(d) Determine the amount you would have to deposit in the case of (b) for the balance to be 250 000 after 10 years.

(e) Explain why the answer in (d) is given by the expression $\frac{250\,000}{1.019^6 \cdot 1.034^4}$.

Problem 4

(a) Determine the present discounted value of 70 million paid 4 years from now when the interest is 9%.

Suppose 70 million is paid after 5 years. Let r be the interest which gives the same present discounted value as in (a).

(b) Do you think that r is bigger or smaller than 9%? – find arguments.

(c) Determine r.

(d) Explain why the answer in (c) can be written as $1.09^{\frac{4}{5}} - 1$.

Problem 5 A payment of 20 million today is supposed to give a return of 9 million 4 years from now and another 14 million 7 years from now.

(a) Suppose the interest is 12%. Determine the present value of the investment.

(b) Do you think the interest has to increase or decrease for the investment to have 0 present value? – find arguments.

(c) Show that the internal rate of return is 2.44%.

(d) Do you think this investment sounds interesting? Give arguments.

Problem 6 A pharmaceutical company plans to test a new drug and then sell the patent. The testing lasts for 5 years and costs 400 million each year. The patent is sold immediately after testing ends.

(a) Suppose the rate of discount is 12%. What should the patent cost for rate of discount to equal the internal rate of return for the cash flow?

(b) The patent is sold for 3.20 billion. Determine the internal rate of return of the cash flow.

Problem 7 We have the cash flow

Year	0	3	5	7	8
Payment	−30	−15	4	11	48

Suppose the interest is 9%.

(a) Calculate the future value of the cash flow after 8 years.

(b) Calculate the present value.

Suppse the interest is 13%.

(c) Calculate the future value of the cash flow after 8 years.

(d) Calculate the present value.

(e) Suppose r is the interest, and that the future value of the cash flow after n years is K_n. Then K_0 in particular is the present value of the cash flow. Explain why

$$K_8 = K_0 \cdot (1+r)^8$$

and check that this is correct in (a-b) and (c-d). Explain why the sign of K_0 is the same as the sign of K_8.

Problem 8

Hege and Kåre are saving. Hege saves 5 300/month. Kåre saves 4 800/month. The yearly nominal interest is 3.6% and the interest is added each month. When they start the saving Kåre already has 200 000 in the account, a present from his dad because he (broadly speaking) hasn't taken up smoking. Hege has nothing.

(a) When is Hege overtaking Kåre?

(b) What happens if the interest is 2.8%?

Problem 9

(a) A company has 63 million in yearly costs. Suppose the cost increases by 2.8% each year. Determine the geometric series which expresses the total cost over a period of 10 years and calculate this total cost.

(b) You deposit 100 NOK today and for every year in 20 years. Suppose the interest is 2% with interest added annually. Determine the geometric series which gives the balance after 20 years and calculate this balance.

Problem 10 Suppose a fixed sum A (the annuity) is paid every year for 25 years with the first payment one year from now. Suppose the interest is 4.2%.

(a) Determine the geometric series which expresses the present value of this cash flow.

(b) Determine the geometric series which expresses the future value K_{25} of this cash flow.

(c) Suppose the present value of the cash flow is 3 million NOK. Use (i) to calculate the annuity A.

(d) Use the annuity you found in (c) to calculate the future value K_{25}.

Answers

Problem 1

(a) Relative change is $r_1 = \frac{899-699}{699} = 28.61\%$. Growth factor is $1 + r_1 = 1.2861$.
(b) Relative change: $r_2 = -22.25\%$. Growth factor: 0.7775.
(c) $(1+r_1)(1+r_2) = 1$.

Problem 2

(a) (i) 71 214.36 (ii) Growth factor: 1.4243. Relative change: 42.43%.
(b) (ii) 71 627.86 (ii) Growth factor: 1.4326. Relative change: 43.26% (iii) 3.66%.

Problem 3

(a) 178 951.20
(b) 229 013.92
(d) 195 349.70

Problem 4

(a) 49.58 million NOK
(c) 7.14%

Problem 5

(a) −7.95 million NOK

Problem 6

(a) 2.541 billion NOK
(b) $r = 23.7\%$ (Hint: You should get the equation $(r+1)^5 - 8r - 1 = 0$. Use e.g. GeoGebra or Wolfram Alpha to draw the graph and read off the root.)

Problem 7

(a) −17.69
(b) −8.88
(c) −41.19
(d) −15.49

Problem 9

(a) In million NOK: $63 \cdot (1 + 1.028 + 1.028 + \cdots + 1.028^9) = 63 \cdot \frac{1.028^{10}-1}{0.028} = 715.61$
(b) $100 \cdot (1.02 + 1.02^2 + \cdots + 1.02^{20}) = 100 \cdot 1.02 \cdot \frac{1.02^{20}-1}{0.02} = 2\,478.33$

Problem 10

(a) $A \cdot \left(\frac{1}{1.042} + \left(\frac{1}{1.042}\right)^2 + \cdots + \left(\frac{1}{1.042}\right)^{25} \right)$
(b) $A \cdot \left(1.042^{24} + 1.042^{23} + \cdots + 1.042 + 1\right)$
(c) $A \cdot 15.297012 = 3$ mill gives $A = \frac{3}{15.297012}$ mill $= 196\,116.73$.
(d) 8 391 009.81