Key Problems

Problem 1.
Use Lagrange's method to find candidates for maximum and/or minimum:

a) $\max / \min f(x,y) = 3x - y$ when $x^2 + 4y^2 = 37$
b) $\max / \min f(x,y) = x^2 + 4y^2$ when $3x - y = 37$
c) $\max / \min f(x,y) = xy$ when $x^2 + 4y^2 = 8$
d) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $xy = 6$
e) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $x^2 + y^2 = 16$
f) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $xy = 4$

Problem 2.
Find the global maximum/minimum, if it exists:

a) $\max / \min f(x,y) = 3x - y$ when $x^2 + 4y^2 = 37$
b) $\max / \min f(x,y) = x^2 + 4y^2$ when $3x - y = 37$
c) $\max / \min f(x,y) = xy$ when $x^2 + 4y^2 = 8$
d) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $xy = 6$
e) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $x^2 + y^2 = 16$
f) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $xy = 4$

Problem 3.
Solve the Lagrange problem: $\max U(x,y) = 0.3 \ln(x - 3) + 0.7 \ln(y - 2)$ when $12x + 5y = 60$. Find the Lagrange multiplicator λ, and give an interpretation of this multiplicator.

Problem 4.
What does a degenerate constraint mean? If you can, give an example of a constraint $g(x,y) = a$ that has an admissible point with degenerate constraint, and a function $f(x,y)$ such that the Lagrange problem $\max f(x,y)$ when $g(x,y) = a$ has a maximum point with degenerate constraint.

Problem 5. *Exam MET11803 12/2015*
We consider the level curve $g(x,y) = 0$, where g is the function $g(x,y) = x^3 + xy + y^2$.

a) Find all point on the level curve with $x = -2$, and determine the tangent line at each point.
b) Find the maximum value of $f(x,y) = x$ when $x^3 + xy + y^2 = 0$.

Problem 6. *Exam MET11803 06/2016*
We consider the Lagrange problem

$$\max / \min f(x,y) = x + 2y - \sqrt{36 - x^2 - 4y^2} \quad \text{when} \quad x^2 + 4y^2 = 36$$

a) Find all points on the level curve $x^2 + 4y^2 = 36$ where the tangent line has slope $y' = 1/2$.
b) Sketch the set $D = \{(x,y) : x^2 + 4y^2 = 36\}$. Is D bounded? What kind of curve is it?
c) Solve the Lagrange problem and find the maximum and minimum values.
d) Solve the new optimization problem we get by changing the constraint to $x^2 + 4y^2 \leq 36$.

Problem 7. *Difficult!*
Solve the Lagrange problem $\max f(x,y) = x + y$ when $x^3 - 3xy + y^3 = 0$. You may assume that the problem has a maximum.

Problem 8.
Problem 7.6.4 - 7.6.6 (norwegian textbook, optional)
Problem 9.32 - 9.34 (norwegian workbook, optional)

Answers to Key Problems

Problem 1.

a) $(x,y;\lambda) = (6,-1/2;1/4),\ (-6,1/2;-1/4)$

b) $(x,y;\lambda) = (12,-1;8)$

c) $(x,y;\lambda) = (2,1;1/4),\ (-2,-1;1/4),\ (2,-1;-1/4),\ (-2,1;-1/4)$

d) $(x,y;\lambda) = (3,2;12),\ (-3,-2;12)$

e) $(x,y;\lambda) = (\pm 2\sqrt{2}, \pm 2\sqrt{2}; 8), (\pm 4,0;0), (0,\pm 4;0)$

f) $(x,y;\lambda) = (2,2;-2), (-2,-2;-2)$

Problem 2.

a) $f_{\text{max}} = 37/2,\ f_{\text{min}} = -37/2$

b) $f_{\text{min}} = 148$ (no maximum)

c) $f_{\text{max}} = 2,\ f_{\text{min}} = -2$

d) $f_{\text{min}} = 72$ (no maximum)

e) $f_{\text{max}} = 64,\ f_{\text{min}} = 0$

f) $f_{\text{max}} = 24$ (no minimum)

Problem 3.

We find the maximum point $(x,y) = (67/20, 99/25)$, and the maximum value $U_{\text{max}} = 1.7\ln(1.4) - 0.6\ln(2)$ with $\lambda = 1/14$. This means that the maximal utility U_{max} would increase with approximately $1/14$ when the constraint is changed from $12x + 5y = 60$ to $12x + 5y = 61$.

Problem 7.

$f_{\text{max}} = 3$