# EBA2911 Mathematics for Business Analytics autumn 2019
## Exercises

*... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.*

R. Lucas

## Lecture 8
### Sec. 6.3.1-3, 5.4-5, 4.7
### Increasing/decreasing functions. Circles, ellipses. Polynomial functions.

Here are recommended exercises from the textbook [SHSC].

Section **6.3** exercise 3
Section **5.4** exercise 1, 3
Section **5.5** exercise 1-6
Section **4.7** exercise 4

### Problems for the exercise session
### Wednesday 2 Oct. from 14 o'clock in B2-085

**Problem 1** Determine the equations of the circles in figure 1.

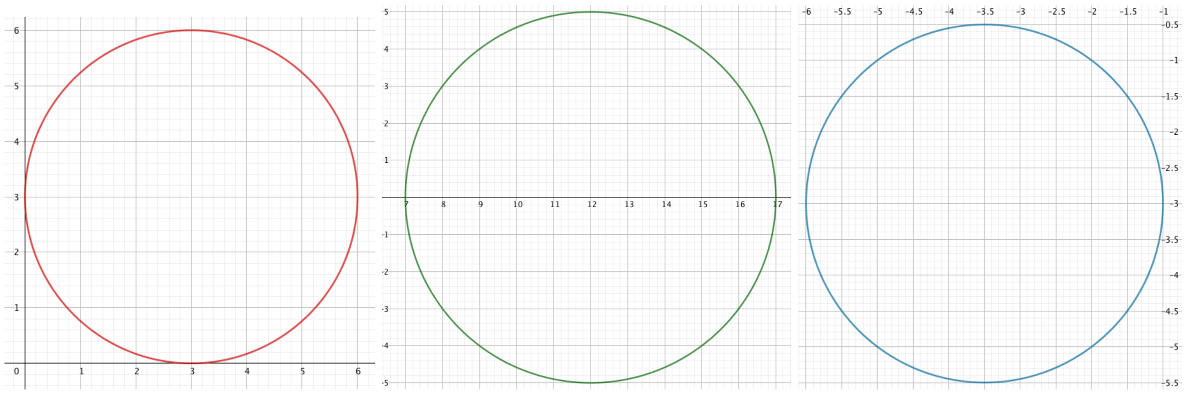

Figure 1: Circles a-c

**Problem 2** Determine the center $S$ and the radius $r$ of the circles.

a) $(x-3)^2+(y-4)^2=5$ b) $(x+1)^2+y^2=3$ c) $(3x-2)^2+(3y-4)^2=9$

d) $x^2+y^2-4x-10y=-25$ e) $x^2+y^2+6x-12y=-44$

f) $25x^2+25y^2-20x-30y=-12$

**Problem 3** Determine the equations of the ellipses in figure 2.

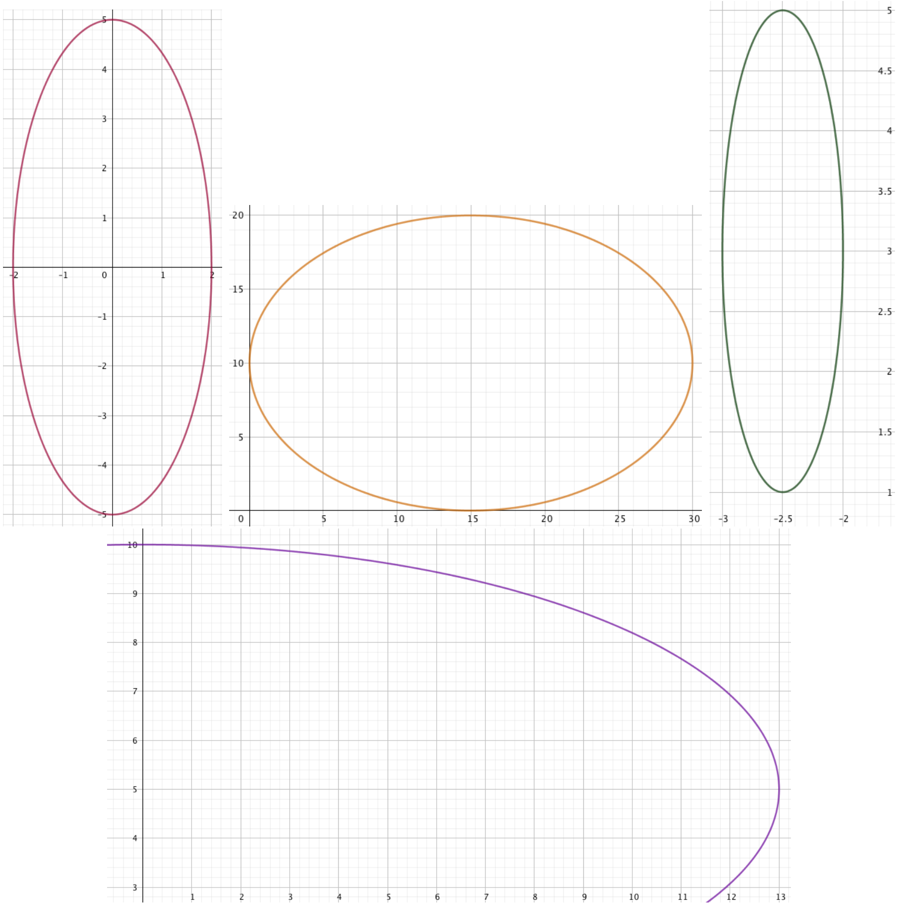

Figure 2: Ellipses a-d

**Problem 4** Determine the center $S$ and the semi-axes of the ellipse. Draw a sketch of the ellipse.

a) $\dfrac{x^2}{9}+\dfrac{y^2}{16}=1$ b) $\dfrac{(x-1)^2}{9}+\dfrac{(y-2)^2}{16}=1$ c) $16(x-1)^2+9(y-2)^2=144$

d) $\dfrac{x^2}{2}+y^2-6y=-8$ e) $9x^2+18x+4y^2=27$ f) $4x^2+9y^2-16x+18y=11$

g) $25x^2+4y^2-100x-40y=-100$

**Problem 5** Give elementary arguments for the statements.
a) $f(x)=x^2$ med $x\geqslant 0$ is strictly increasing.
b) $f(x)=\sqrt{x}$ is strictly increasing.
c) $f(x)=\frac{1}{x}$ with $x>0$ is strictly decreasing.

**Problem 6** Determine the intersection points of
a) the line $3x+2y=12$ and the line $-3x+2y=-6$
b) the line $2x+y=6$ and the ellipse in Problem 4a

**Problem 7** Determine which expressions (below) and graphs (in figure 3) which belong together.

- $x^4-8x^3+24x^2-32x+\dfrac{161}{10}$
- $\dfrac{x^5}{10}-\dfrac{3x^4}{2}+\dfrac{17x^3}{2}-\dfrac{45x^2}{2}+\dfrac{137x}{5}-10$
- $-x^3+6x^2-11x+7$
- $x^4-10x^3+35x^2-50x+26$

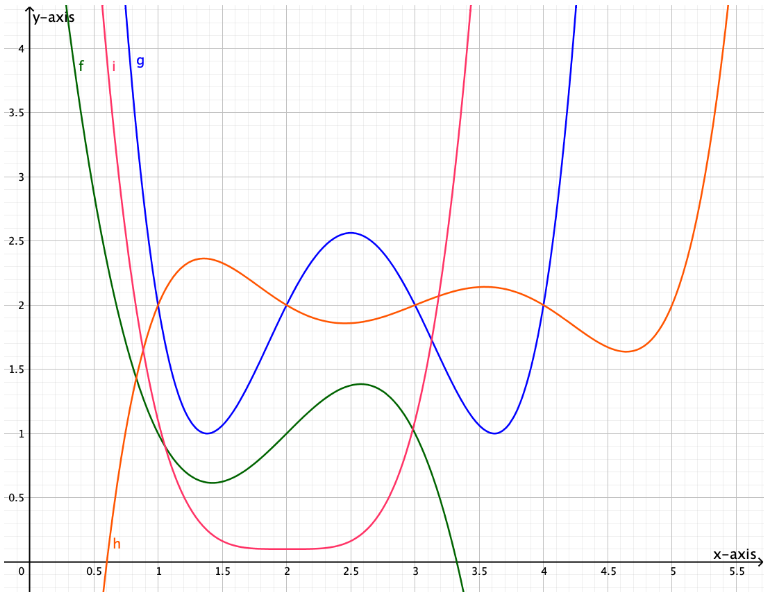


Figure 3: The graphs of four polynomial functions

## Answers

### Problem 1

a) $(x-3)^2+(y-3)^2=9$ b) $(x-12)^2+y^2=25$ c) $(x+3{,}5)^2+(y+3)^2=6{,}25$

### Problem 2

a) $S=(3,4)$, $r=\sqrt{5}$ b) $S=(-1,0)$, $r=\sqrt{3}$ c) $S=(\frac{2}{3},\frac{4}{3})$, $r=1$

d) $S=(2,5)$, $r=2$ e) $S=(-3,6)$, $r=1$ f) $S=(\frac{2}{5},\frac{3}{5})$, $r=\frac{1}{5}$

### Problem 3

a) $\frac{x^2}{4}+\frac{y^2}{25}=1$ b) $\frac{(x-15)^2}{225}+\frac{(y-10)^2}{100}=1$

c) $4(x+2{,}5)^2+\frac{(y-3)^2}{4}=1$ d) $\frac{x^2}{169}+\frac{(y-5)^2}{25}=1$

### Problem 4

a) $S=(0,0)$, semi-axes $a=3$, $b=4$ b) $S=(1,2)$, semi-axes $a=3$, $b=4$

c) $S=(1,2)$, semi-axes $a=3$, $b=4$ d) $S=(0,3)$, semi-axes $a=\sqrt{2}$, $b=1$

e) $S=(-1,0)$, semi-axes $a=2$, $b=3$ f) $S=(2,-1)$, semi-axes $a=3$, $b=2$

g) $S=(2,5)$, semi-axes $a=2$, $b=5$

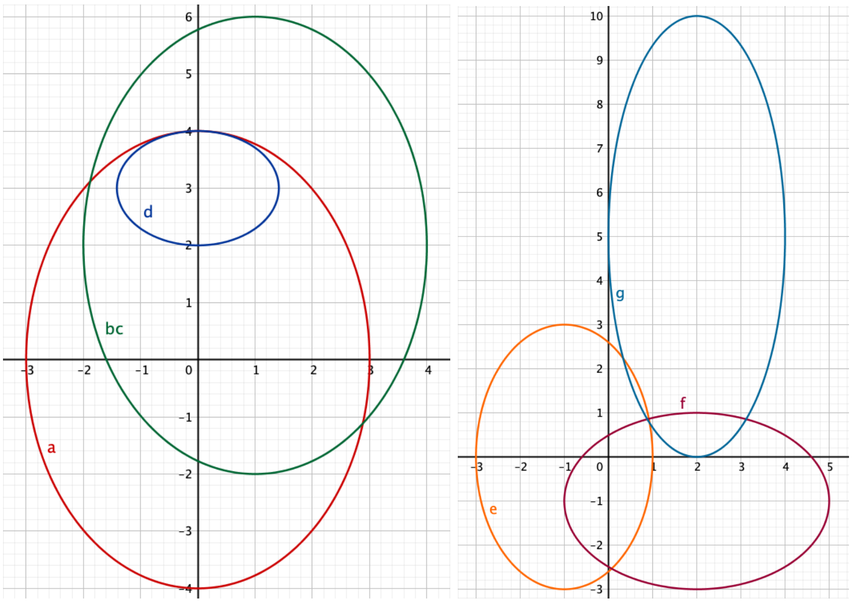


Figure 4: Ellipses a-d and e-g

**Problem 5**

a) Suppose $0 \leqslant x_1 < x_2$. Then $x_2 = x_1 + k$ for a positive constant $k$. Then $f(x_2) = (x_1 + k)^2 = x_1^2 + 2kx_1 + k^2$. We have $2kx_1 + k^2 = k(2x_1 + k)$ which is a product of two positive numbers, hence a positive number. Then $f(x_1) = x_1^2 < x_1^2 + 2kx_1 + k^2 = f(x_2)$ and $f(x) = x^2$ for $x \geqslant 0$ is strictly increasing.

b) We divide each side of the inequality $x_1 < x_2$ with the positive number $x_2$ and get the inequality $\frac{x_1}{x_2} < 1$. The square root of a number which is less than 1 is itself less than 1, i.e. $\sqrt{\frac{x_1}{x_2}} < 1$. But $\sqrt{\frac{x_1}{x_2}} = \frac{\sqrt{x_1}}{\sqrt{x_2}}$. We get the inequality $\frac{\sqrt{x_1}}{\sqrt{x_2}} < 1$ and when we multiply each side with the positive number $\sqrt{x_2}$ we get the inequality $f(x_1) = \sqrt{x_1} < \sqrt{x_2} = f(x_2)$. Hence $f(x) = \sqrt{x}$ strictly increasing.

c) We divide each side of the inequality $x_1 < x_2$ with the positive number $x_2$ and get the equivalent inequality $\frac{x_1}{x_2} < 1$. Then we divide this inequality by the positive number $x_1$ and get $f(x_2) = \frac{1}{x_2} < \frac{1}{x_1} = f(x_1)$. Hence $f(x) = \dfrac{1}{x}$ for $x > 0$ is strictly decreasing.

**Problem 6**

a) $(3, \frac{3}{2})$

b) $(3, 0)$ and $(\frac{15}{13}, \frac{48}{13})$

**Problem 7**

- $f(x) = -x^3 + 6x^2 - 11x + 7$
- $g(x) = x^4 - 10x^3 + 35x^2 - 50x + 26$
- $h(x) = \dfrac{x^5}{10} - \dfrac{3x^4}{2} + \dfrac{17x^3}{2} - \dfrac{45x^2}{2} + \dfrac{137x}{5} - 10$
- $i(x) = x^4 - 8x^3 + 24x^2 - 32x + \dfrac{161}{10}$