EBA2911 Mathematics for Business Analytics autumn 2019

Exercises

… if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

R. Lucas

Lecture 9

Sec. 4.7, 7.9, 7.8, 5.2

Rational functions and asymptotes. Continuity. Composing functions.

Here are recommended exercises from the textbook [SHSC].

Section **4.7** exercise 4
Section **7.9** exercise 1-5
Section **7.8** exercise 1-5
Section **5.2** exercise 2a, 3, 4

Multiple choice exam spring 2018 (translated)

Problem 8 The function

$$f(x) = \frac{2x^2 + 5x - 7}{x^2 - 2x + 3}$$

Which statement is true?

A) The function has only vertical asymptotes.
B) The function has only horizontal asymptotes.
C) The function has one vertical and one horizontal asymptote.
D) The function has two vertical and one horizontal asymptote.
E) I choose not to answer this question.

Solution

Multiple choice exam spring 2018, Problem 8
Note that $x^2 - 2x + 3 = (x-1)^2 + 2$ which is never equal to 0. Hence there are no vertical asymptotes. This gives B.

We could also find the vertical asymptote(s) by polynomial division. We get

$$\begin{array}{l} \big(\quad 2x^2 + 5x \quad - 7\big) : \big(x^2 - 2x + 3\big) = 2 + \dfrac{9x - 13}{x^2 - 2x + 3} \\ \underline{-2x^2 + 4x \quad - 6} \\ \qquad\quad 9x - 13 \end{array}$$

Since

$$\frac{9x - 13}{x^2 - 2x + 3} = \frac{\frac{9}{x} - \frac{13}{x^2}}{1 - \frac{2}{x} + \frac{3}{x^2}}$$

approaches $\dfrac{0}{1} = 0$ when x (or $-x$) grows without bounds ($x \to \pm\infty$), it follows that

$$\frac{2x^2 + 5x - 7}{x^2 - 2x + 3}$$

approaches 2 when x (or $-x$) grows without bounds. So the horizontal line $y = 2$ is a horizontal asymptote for $f(x)$.