EBA 2911, lecture 12, 27 Oct. 2020, Runar Ile

Plan Do some problems from the midterm

1. Financial math : Prob. 1 and 10
2. Ellipses. Prob 7
3. Inverse functions. Prob. 9b

1. Financial math

Prob. 1a

$$\frac{10\,000}{1.01^{36}} + \frac{10\,000}{1.01^{37}} + \cdots + \frac{10\,000}{1.01^{215}}$$

$\cdot 1.01$

$\cdot \frac{1}{1.01}$

Geometric series 'backwards' : $a_1 = \frac{10\,000}{1.01^{215}}$

$$k = 1.01$$

# of terms : $n = 215 - 35 = 180$

Formula for the sum :

$$a_1 \cdot \frac{k^n - 1}{k-1}$$

$$= \frac{10\,000}{1.01^{215}} \cdot \frac{1.01^{180} - 1}{0{,}01} = \underline{\underline{588179{,}46}}$$

The sum gives the present value of a regular cash flow where:

i) 10 000 is paid every month, for 15 years $\left(\frac{180}{12} = 15 \right)$

ii) The first payment is 3 years from now. $\left(\frac{36}{12} = 3 \right)$

iii) 12% nominal interest, monthly compounding

1b) $$\frac{10\,000}{e^{0.36}} + \frac{10\,000}{e^{0.37}} + \cdots + \frac{10\,000}{e^{2.15}} \qquad (*)$$

Because $e^{0.36} = (e^{0.01})^{36}$, $e^{0.37} = (e^{0.01})^{37}$, ...

the sum $(*)$ is the present value of a regular cash flow with (i) and (ii) as in 1a, but with

iii') 12% nominal interest and continuous compounding
(because then the monthly growth factor is $(e^{0.12})^{\frac{1}{12}} = e^{0.01}$)

Prob 10

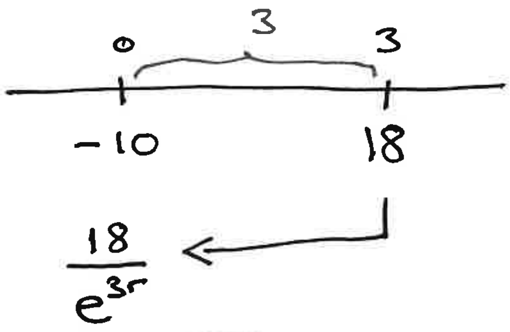
Year	m	n
Payment	$-A$	B

$m, n, A, B \geq 0$

with continuous compounding.

Put r = internal rate of return (IRR)

a i)



The sum = tot. present value . which is supposed to be 0 when r = IRR .

Get equation $-10 + \frac{18}{e^{3r}} = 0$

$$\frac{18}{e^{3r}} = 10 \quad \Big| \cdot e^{3r}$$

$$18 = 10e^{3r} \quad \Big| : 10$$

$$1.8 = \frac{18}{10} = e^{3r}$$

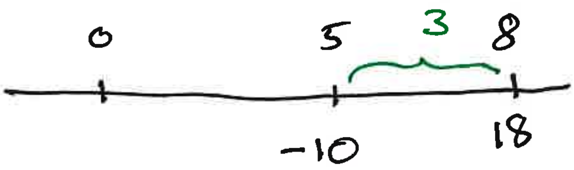
Insert both sides into $\ln(-)$.

$$\ln(1.8) = \ln(e^{3r}) = 3r \cdot \ln(e) = 3r \cdot 1 = 3r$$

so $3r = \ln(1.8)$

$$r = \frac{\ln(1.8)}{3} = 19.593\,\%$$

a ii)



$$\frac{-10}{e^{5r}}$$

$$\frac{18}{e^{8r}}$$

$$= 0$$

Get eq:

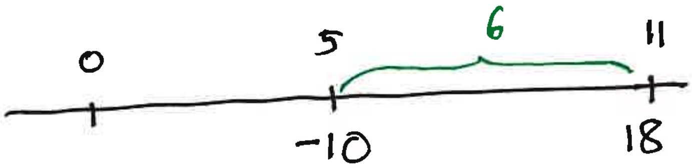
$$\frac{18}{e^{8r}} = \frac{10}{e^{5r}} \quad \Big| \cdot \frac{e^{8r}}{10}$$

$$\frac{18}{10} = \frac{e^{8r}}{e^{5r}} = e^{8r-5r}$$

$$= e^{3r}$$

same eq. as in (i).

a iii)



$$-\frac{10}{e^{5r}}$$

$$\frac{18}{e^{11r}}$$

$$= 0$$

Get eq.

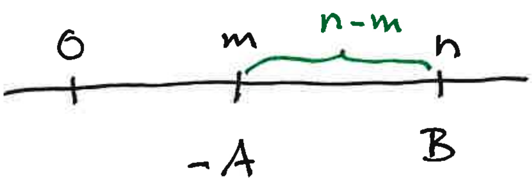
$$\frac{18}{e^{11r}} = \frac{10}{e^{5r}} \quad \Big| \cdot \frac{e^{11r}}{10}$$

$$1.8 = \frac{18}{10} = \frac{e^{11r}}{e^{5r}} = e^{6r}$$

so $\quad 6r = \ln(1.8)$

and $\quad r = \dfrac{\ln(1.8)}{6} = 9.796\,\%$

10b)



$$\frac{-A}{e^{mr}}$$

$$\frac{B}{e^{nr}}$$

$$= 0$$

Get eq:

$$\frac{B}{e^{nr}} = \frac{A}{e^{mr}} \quad \Big| \cdot \frac{e^{nr}}{A} \quad (A>0)$$

$$\frac{B}{A} = \frac{e^{nr}}{e^{mr}} = e^{nr-mr} = e^{(n-m)r}$$

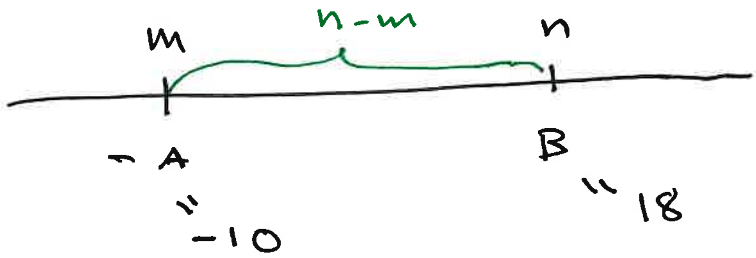
$(B>0)$ $\quad \ln\left(\frac{B}{A}\right) = (n-m)r \quad \Big| : (n-m) \neq 0$

$$r = \frac{\ln\left(\frac{B}{A}\right)}{n-m}$$

10c) In the formula $r = \dfrac{\ln(1.8)}{n-m}$

$n-m$ is the only changing part

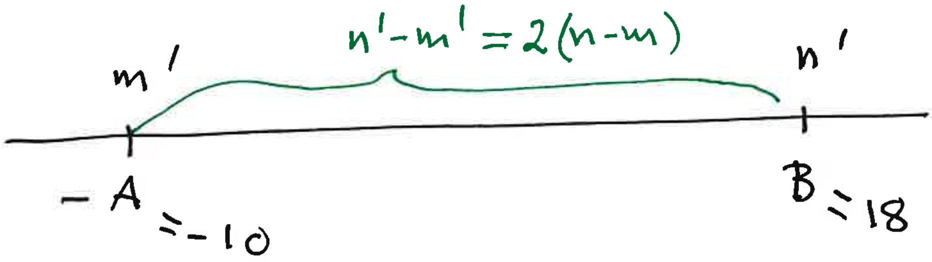
$n-m$ is the length of the time interval between the payments



Suppose now that payments $-A = -10$ at time m'

and $B = 18$ at time n'.

such that $n' - m' = 2(n-m)$



Then the new IRR $r' = \dfrac{\ln(1.8)}{n'-m'} = \dfrac{\ln(1.8)}{2(n-m)}$

$$= \frac{1}{2} \cdot \frac{\ln(1.8)}{(n-m)} = \frac{1}{2} \cdot r$$

$$= \underline{\underline{\frac{r}{2}}}$$

Hence twice as long period between payments halves the IRR.

2. Ellipses

Prob. 7a If $C = (x_0, y_0)$ is the centre of an ellipse with horizontal semi-axis a and vertical — " — b

then the ellipse is given as the solutions of the equation

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

We see that $C = (5, 3)$ and

a = distance between C and $(10, 3)$

$= 10 - 5 = \underline{5}$

b = distance between C and $(5, 6)$

$= 6 - 3 = \underline{3}$

so the ellipse equation is

$$\frac{(x-5)^2}{25} + \frac{(y-3)^2}{9} = 1$$

b) $P = (10, 3)$ and L is the line throug P with slope $= -0.3$.

L is given by the point-slope formula

$$y - 3 = -0.3 \cdot (x - 10)$$

$$y = -0.3x + 3 + 3 = \underline{-0.3x + 6}$$

Substitute the y in the ellipse eq. with this expression:

$$\frac{(x-5)^2}{25} + \frac{(\overbrace{-0.3x+6-3}^{-0.3x+3})^2}{9} = 1 \quad \Big| \cdot 25 \cdot 9$$

$$9(x^2 - 10x + 25) + 25(0.09x^2 - 1.8x + 9) = 225$$

$$9x^2 - 90x + 225 + 2.25x^2 - 45x + 225 = 225$$

$$11.25x^2 - 135x = -225 \quad |:11.25$$

$$x^2 - 12x = -20$$

$:2$

$$(x-6)^2 = -20 + 36 = 16$$

$x - 6 = 4$ or $x - 6 = -4$

$\underline{x = 10}$ or $\underline{x = 2}$

(given) (the new)

$$y = -0.3 \cdot 2 + 6 = \underline{5.4}$$

so the second intersection point is $\underline{\underline{(2, 5.4)}}$

3. Inverse functions

Prob. 9b $f(x) = 2 \cdot \ln(x+3) - 1$, $D_f = \langle -3, \rightarrow \rangle$

supposed to find the inverse function $g(x)$ with D_g and R_g.

① We solve the equation $y = 2\ln(x+3) - 1$ for x

add 1 on each side

$$y + 1 = 2\ln(x+3)$$

divide by 2 on e. s.

$$0.5y + 0.5 = \frac{y+1}{2} = \ln(x+3)$$

Insert the l.h.s and the r.h.s into $e^{(\,)}$

$$e^{0.5y+0.5} = e^{\ln(x+3)} = x+3$$

subtract 3 from each side

$$e^{0.5y+0.5} - 3 = x$$

② switch x and y:

$$g(x) = \underline{\underline{e^{0.5x+0.5} - 3}}$$

③ $R_g \overset{\text{always}}{=} D_f = \underline{\underline{\langle -3, \rightarrow \rangle}}$

and $D_g \overset{\text{always}}{=} R_f$ - has to be determined.

We have $\underbrace{2\ln(x+3)}_{\substack{\downarrow\, x+3\to 0^+ \\ -\infty}} - 1 \xrightarrow[x\to -3^+]{} -\infty$

Moreover, $\underbrace{2\ln(x+3)}_{\text{increasing without bounds if } x+3\to\infty} - 1 \xrightarrow[x\to\infty]{} +\infty$

Hence $D_g = V_f$ = <u>the whole number line</u>