EBA 2911, lecture 2, 26 Aug. 2020, Runar Ile

Plan.
1. Relative change and rate of change
2. Powers
3. Interest
4. Present value of cash flow.

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1. Relative change and rate of change

$$\text{Relative change} = \frac{\text{new value} - \text{old value}}{\text{old value}}$$

Recall: $\% = \frac{1}{100} = 0.01$

$$3\% = 3 \cdot \frac{1}{100} = \frac{3}{100} = 0.03$$

Ex Kåre's hourly wage increased from 163 kr to 181 kr.

The relative change was

$$\frac{181\text{ kr} - 163\text{kr}}{163\text{kr}} = \frac{18}{163} = 11.0\%$$

$$\text{Rate of change} = 1 + \text{relative change}$$

Ex The rate of change in Kåre's hourly wage is $1 + 0.11 = 1.11$

Problem Last year Kåre earned 54000 with 163/hour. If he works as much this year as last year how much would he earn (with the new wage)?

Solution $54000 \cdot 1.11 = \underline{\underline{59940}}$

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2. Powers

$$1.11^3 = 1.11 \cdot 1.11 \cdot 1.11$$

$$1.11^{-3} = \frac{1}{1.11^3}$$

$$1.11^{\frac{2}{3}} = \sqrt[3]{1.11^2}$$

For integers m, n with $n > 0$ and a number $a \geq 0$, then

$$a^{\frac{m}{n}} \overset{\text{definition}}{=} \sqrt[n]{a^m}$$

Problem Calculate $1.11^{\sqrt{2}}$ on your calculator!

(answer: 1.159035....)

Answer 1.11 $\boxed{y^x}$ 2 $\boxed{\sqrt{x}}$ $\boxed{=}$

Same base:

$$2^{1.5} \cdot 2^{3.8} = 2^{1.5+3.8} = 2^{5.3}$$

Same exponent:

EX $2^{4} \cdot 3^{4} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3$

$= (2 \cdot 3)^4 = 6^{4}$

EX $\sqrt{2} \cdot \sqrt{3} = 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = (2 \cdot 3)^{\frac{1}{2}} = \sqrt{6}$

Pattern: $a^r \cdot b^r = (ab)^r$

Problem: Calculate 2^{-1} on the calculator.

Solution 1: 2 $\boxed{y^x}$ 1 $\boxed{+/-}$ $\boxed{=}$

Solution 2: 2 $\boxed{1/x}$ $\boxed{=}$ (reason: $2^{-1} = \frac{1}{2}$)

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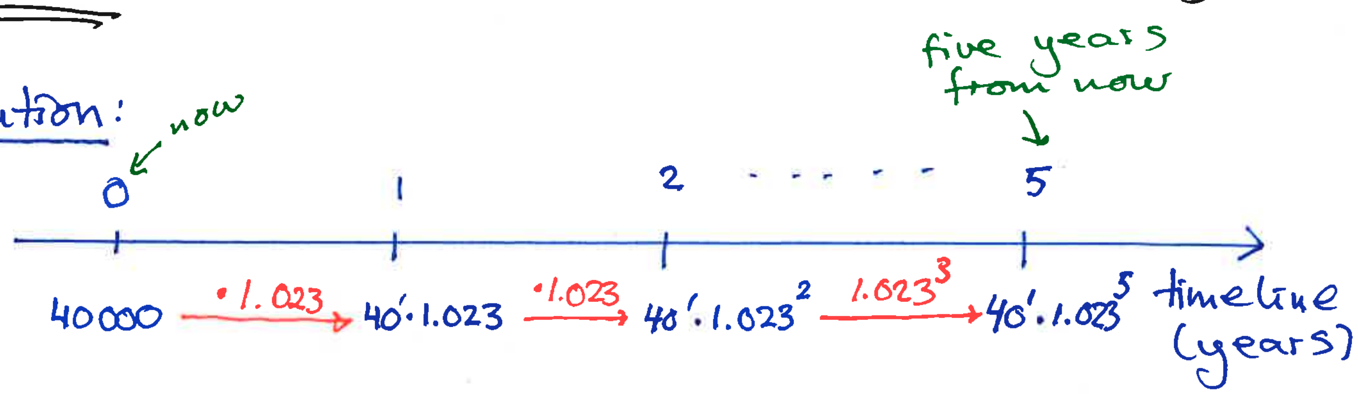
3. Interest

EX You deposit 40 000 into an account earning 2.3% annual interest. Interest is added after each year (annual compounding of interest). After a year the balance (what's in the account) is given as $40\,000 + 40\,000 \cdot 2.3\%$

$$= 40\,000 \cdot \underbrace{(1 + 0.023)}_{\text{growth factor}} = 40\,920.00$$

Problem What is the balance after 5 years?

Solution:



$$\underline{\underline{40000 \cdot 1.023^5}} = \underline{\underline{44\,816.52}}$$

EX You deposit 40000 with 2.3% nominal annual interest, but with quarterly compounding of interest.

The growth factor for one period (= 3 mnths) is

$$1 + \frac{2.3\%}{4} = 1 + \underbrace{0.575\%}_{\text{the interest rate for one period (= 3 months)}} = 1.00575$$

After 1 year the balance is

$$40\,000 \cdot 1.00575^4 = \underline{\underline{40927.97}}$$

The annual growth factor is

$$1.00575^4 = 1.023199$$

The effective annual interest is

$$1.00575^4 - 1 = 0.023199$$
$$= 2{,}3199\,\%$$

nominal interest

Pattern $\quad B = B_0 \cdot \left(1+\frac{r}{n}\right)^{m}$ – number of periods

B: balance after m periods

B_0: deposit (principal)

n: interest periods per year

Effective interest $\quad r_{eff} = \left(1+\frac{r}{n}\right)^{n} - 1$

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4. Present value of cash flow

Let K_0 be some investment / deposit / payment today

The future value K_n of K_0 in n years (or more generally n periods) with interest r is

$$K_n = K_0 (1+r)^n$$

solve for K_0

The opposite: Suppose K_n will be paid in n years (periods) from now with period interest r.

Then the present value K_0 of K_n is given as

$$K_0 = \frac{K_n}{(1+r)^n}$$

Problem 30 mill. is paid 5 years from now with 8% (annual) interest. Determine the present value.

Solution $$K_0 = \frac{30 \text{ mill.}}{1.08^5} = \underline{\underline{20.42 \text{ mill}}}$$

"How much do you have to deposit today to have 30 mill 5 years from now if the interest is 8%?"

Cash flow

Ex You pay 20 mill today, and will get paid back

6 mill. after 3 years
7 mill. after 4 years
8 mill after 5 years with 8% interest

year	0	3	4	5
payment	-20	6	7	8

The present value of the cash flow is the sum of the present values of the payments:

$$-20 + \frac{6}{1.08^3} + \frac{7}{1.08^4} + \frac{8}{1.08^5} = \underline{\underline{-4.65}}$$

(bad investment!)

The interest which makes the present value of the cash flow equal to 0 is called the internal rate of return.

(„internrenten")

- hard to find in general.

Homework: Determine the internal rate of return for the cash flow in the example. (answer: 1.1197%)