EBA2911, lecture 6, 16 Sept. 2020, Runar Ile

Plan
1. Polynomial division and factorisation
2. Rational and radical equations
3. Inequalities

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1. Polynomial division and factorisation

Want to divide the polynomial $f(x)$
with a polynomial $g(x)$
and get a polynomial $q(x)$
with a remainder $r(x)$

$$g(x)\cdot \Big| \quad \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \qquad \text{with } \deg(r(x)) < \deg(g(x))$$

gives $f(x) = q(x)\cdot g(x) + r(x)$

Ex $f(x) = 3x^2 + 2x + 1$ and $g(x) = x - 2$

$$
\begin{array}{l}
(\boxed{3x^2} + 2x + 1) : (\boxed{x} - 2) = \overset{3x^2 : x}{\overset{\|}{3x}} + \overset{8x : x}{\overset{\|}{8}} + \dfrac{17}{x-2} \\
\underline{-(3x^2 - 6x \qquad)} \qquad \cdot(x-2) \\
\qquad\quad \boxed{8x} + 1 \\
\qquad \underline{-(8x - 16)} \qquad \cdot(x-2) \\
\qquad\qquad\quad \boxed{17} \qquad :(x-2)
\end{array}
$$

is called the remainder

So $q(x) = 3x + 8$
and $r(x) = 17$.

$\underline{\underline{\text{check}}}$:
$$\left(\underbrace{3x+8}+\overbrace{\frac{17}{x-2}}\right)\cdot\overbrace{(x-2)}$$

$$= (3x+8)(x-2) + \frac{17}{\cancel{x-2}}\cdot\cancel{(x-2)}$$

$$= 3x^2+8x-6x-16+17$$

$$= 3x^2+2x+1 = f(x) \qquad \text{– so ok!}$$

Two applications of polynomial division

A) To find asymptotes of rational functions

EX
$$\frac{3x^2+2x+1}{x-2} = 3x+8+\frac{17}{(x-2)}$$

has a vertical asymptote: the line $x=2$

and a non-vertical asymptote: the line $y=3x+8$

B) To factorise a polynomial as a product of degree 1 (linear) polynomials.

EX Factorise $x^3-4x^2-11x+30$ into linear factors.

Solution Three steps.

Step I Guess an integer root (zero)

[Note: has to divide 30]

I try $x=-3$: $(-3)^3-4\cdot(-3)^2-11\cdot(-3)+30$

$= -27-36+33+30 = 0$

Then $(x-(-3)) = (x+3)$ is a factor!

Step 2 Use polynomial division to find a polynomial of lower degree:

$$
\begin{array}{l}
(\boxed{x^3} - 4x^2 - 11x + 30) : (\boxed{x} + 3) = x^2 - 7x + 10 \\
-(x^3 + 3x^2 \qquad\qquad) \xleftarrow{\cdot (x+3)} \\
\hline
\qquad \boxed{-7x^2} - 11x + 30 \\
\quad -(-7x^2 - 21x \qquad) \xleftarrow{\cdot (x+3)} \\
\hline
\qquad\qquad \boxed{10x} + 30 \\
\qquad\qquad -(10x + 30) \xleftarrow{\cdot (x+3)} \\
\hline
\qquad\qquad\qquad 0 \text{ remainder}
\end{array}
$$

This means $x^3 - 4x^2 - 11x + 30 = (x^2 - 7x + 10)(x+3)$

Step 3 We find the roots (zeros) of $x^2 - 7x + 10$.

They are $x = 2$, $x = 5$.

So $x^2 - 7x + 10 = (x-2)(x-5)$

Then: $x^3 - 4x^2 - 11x + 30 = \underline{\underline{(x-2)(x-5)(x+3)}}$

Note 1 Not always possible to factorise

Ex $x^2 + 5$ has no roots!

$x^2 + 2x + 3$ has no roots! since $b^2 - 4ac$
$= 2^2 - 4 \cdot 1 \cdot 3 = 4 - 12 < 0$

Note 2 It can be difficult to guess a root
- it doesn't have to be an integer

2. Rational- and radical equations

A rational equation: $\frac{p(x)}{q(x)} = 0$

where $p(x)$ and $q(x)$ are polynomials

Ex $\frac{x+1}{(x-1)(x+3)} = 0$ then $x+1 = 0$

and $(x-1)(x+3) \neq 0$

i.e. $x \neq 1$, $x \neq -3$

Ex $\frac{x+1}{(x-1)(x+3)} = 2$

subtract 2 from b.s.

$$\frac{x+1}{(x-1)(x+3)} - 2 = 0$$

multiply -2 with $\frac{(x-1)(x+3)}{(x-1)(x+3)}$ (which is 1)

$$\frac{x+1}{(x-1)(x+3)} - 2 \cdot \frac{(x-1)(x+3)}{(x-1)(x+3)} = 0$$

$$\frac{x+1 - 2(x-1)(x+3)}{(x-1)(x+3)} = 0$$

$$\frac{x+1 - 2(x^2+2x-3)}{(x-1)(x+3)} = 0$$

$$\frac{x+1-2x^2-4x+6}{(x-1)(x+3)} = 0$$

$$\frac{-2x^2-3x+7}{(x-1)(x+3)} = 0$$

that is $-2x^2-3x+7 = 0$

with $x \neq 1$, $x \neq -3$

which you can solve.

Radical equations

- the unknown is under a root!

Ex $2\sqrt{x+1} = x-2$

square both sides

$$4(x+1) = x^2-4x+4$$

$$4x+\cancel{4} = x^2-4x+\cancel{4}$$

$$x^2-8x = 0$$

$$x(x-8) = 0$$

so $\underline{x=0}$ or $\underline{x=8}$

Note Not all of these need to be solutions of the original eq.

We have to test the candidates:

$\underline{x=0}$ l.h.s. $2\cdot\sqrt{0+1} = 2\cdot\sqrt{1} = 2\cdot 1 = 2$
r.h.s. $0-2 = -2$

not equal! so $x=0$ is not a solution

$\underline{x=8}$ l.h.s. $2\cdot\sqrt{8+1} = 2\sqrt{9} = 2\cdot 3 = 6$
r.h.s. $8-2 = 6$

equal! so $x=8$ is the only solution.

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3. Inequalities

$-2 < -1$ read: 'minus two is less than minus one'

$\frac{1}{9} > \frac{1}{12}$ read: 'one ninth is bigger than one twelfth'

Also: $\leqslant$ and $\geqslant$

An inequality is claim that one expression (number) is less than, bigger than... another expression (number)

The solutions of an inequality are those values of x which make the claim true.

Ex. $x-1 \geqslant 2$ is a claim
- is true for $x=5$ since $5-1 \geqslant 2$
- is not true for $x=2$ since $2-1 \geqslant 2$ is not true!

The solutions of the inequality are all the values of x such that $\underline{\underline{x \geq 3}}$.

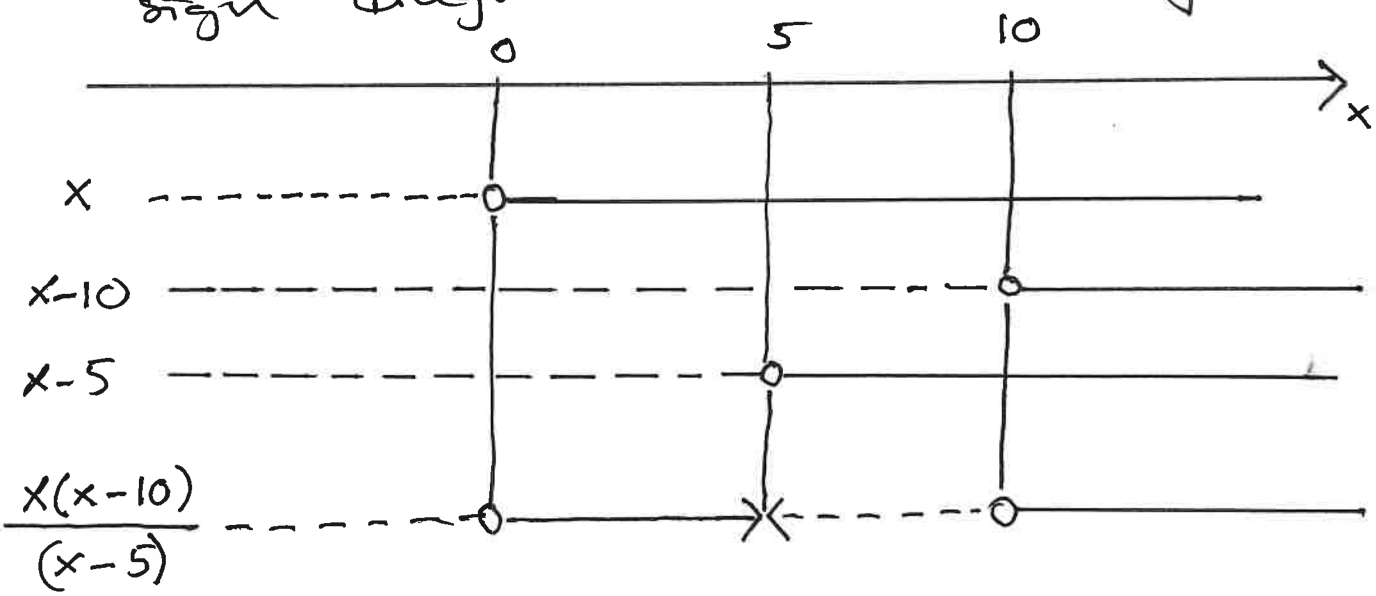
– an infinite set of numbers!

We can also write $\underline{\underline{x \in [3, \infty\rangle}}$

$\underline{\underline{x \in [3, \rightarrow\rangle}}$

<u>Ex</u> Solve the inequality $\dfrac{x(x-10)}{(x-5)} \geq 0$

Solution: Since we have 0 on the r.h.s. and a factorised fraction we can apply the sign diagram immedeately



that is $\underline{\underline{0 \leq x < 5 \text{ or } x \geq 10}}$

another way of writing: $\underline{\underline{x \in [0, 5\rangle \text{ or } x \in [10, \infty\rangle}}$