EBA2911 Mathematics for Business Analytics
autumn 2020
Exercises

> *... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing. I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.*
>
> R. Lucas

Lecture 16

on Wednesday 18 Nov. at 10-11.45 streaming from B2-060
Sec. 7.7, 7.4-5: Elasticity. Linearisation. Taylor polynomials.

Section **7.7** exercise 1-3
Section **7.4** exercise 1-3
Section **7.5** exercise 1-3

Multiple choice exam 2018a, problem 10, 13
Multiple choice exam 2019s, problem 11
Multiple choice exam 2019a, problem 11

Problems for the exercise session Wednesday 18 Nov. at 12-15 on Zoom (it is possible to sit in CU1-067)

Problem 1 Let p be the price of a commodity and $D(p)$ the demand for the commodity with price p (so $D(p)$ is the number of sold units). Determine the relative change of price, the relative change of demand and the (average) elasticity of the demand with respect to price. Determine if the demand is elastic, inelastic, or unit elastic.
a) $D(30) = 40$ and $D(30.5) = 39$
b) $D(20) = 101$ and $D(21) = 100.95$
c) $D(10) = 24.648$ and $D(10.01) = 24.623$

Problem 2 Let p be the price of a commodity and $D(p)$ the demand for the commodity with price p (so $D(p)$ is the number of sold units). Determine the (momentary) elasticity $\varepsilon(p) = \mathrm{El}_p(D(p))$ of the demand with respect to price. Determine the price p such that the demand is elastic, inelastic, and unit elastic.
a) $D(p) = 100 - 2p$ med $0 < p < 50$
b) $D(p) = 100 + \frac{20}{p}$ med $p \geqslant 1$
c) $D(p) = 67e^{-0.1p}$ med $p > 0$
d) $D(p) = 100 + \frac{900}{p^2}$ med $p \geqslant 1$
e) $D(p) = 53e^{-0.02p^2}$ med $p > 0$

Problem 3
a) Determine the Taylor polynomials $P_1(x), \dots, P_4(x)$ of degree $1-4$ of the function $f(x) = e^x$ at 0.
b) Compute $P_1(1), \dots, P_4(1)$ and compute how good approximations these values give to $f(1) = e$.

Problem 4
a) Determine the Taylor polynomials $P_1(x), \dots, P_4(x)$ of degree $1-4$ of the function $f(x) = \ln(x)$ at 1.
b) Compute $P_1(2), \dots, P_4(2)$ and compute how good approximations these values give to $f(2) = \ln(2)$.

Problem 5 We have a function $f(x)$ with $f(50) = 100$, $f'(50) = 1$ and $f''(50) = -0.4$.
a) Determine the Taylor polynomial $P_2(x)$ of $f(x)$ at 50.
b) Use $P_2(x)$ to give an approximate value for $f(52)$.

Problem 6 Let $P_1(x), P_2(x), P_3(x)$ be the three first Taylor polynomials in problem 3. Compute the limit values.

$$\text{a)}\ \lim_{x\to 0}\frac{e^x - P_1(x)}{x^2} \qquad \text{b)}\ \lim_{x\to 0}\frac{e^x - P_2(x)}{x^3} \qquad \text{c)}\ \lim_{x\to 0}\frac{e^x - P_3(x)}{x^4}$$

Problem 7 Let $P_1(x), P_2(x), P_3(x)$ be the three first Taylor polynomials in problem 4. Compute the limit values.

$$\text{a)}\ \lim_{x\to 1}\frac{\ln(x) - P_1(x)}{(x-1)^2} \qquad \text{b)}\ \lim_{x\to 1}\frac{\ln(x) - P_2(x)}{(x-1)^3} \qquad \text{c)}\ \lim_{x\to 1}\frac{\ln(x) - P_3(x)}{(x-1)^4}$$

Problem 8 (Multiple choice exam 2017s, problem 12)
We consider the price elasticity $\varepsilon = \varepsilon(p)$ of a commodity with demand function $D(p) = 120 - 8p$. Then:
(A) $\varepsilon > -1$ for $p = 7.5$
(B) $\varepsilon > -1$ for $p < 7.5$
(C) $\varepsilon > -1$ for $p > 7.5$
(D) $\varepsilon > -1$ for all values of p
(E) I choose not to answer this problem.

Problem 9 (Multiple choice exam 2016a, problem 12)
Demand for a commodity is given as $D(p) = 110 - 5p$. Then the elasticity $\varepsilon(p) = -1$ for:
(A) $p = 7$
(B) $p = 11$
(C) $p = \frac{16}{5}$
(D) $p = 22$
(E) I choose not to answer this problem.

Answers

Problem 1

a) the relative change of price is $\frac{0.5}{30}$, the relative change of demand is $\frac{-1}{40}$ and the elasticity is -1.5, i.e. elastic demand

b) the relative change of price is $\frac{1}{20}$, the relative change of demand is $\frac{-0.05}{101}$ and the elasticity is -0.0099, i.e. inelastic demand

c) the relative change of price is 0.001, the relative change of demand is -0.001014 and the elasticity is -1.014, i.e. elastic demand

Problem 2

a) $\varepsilon(p) = \frac{-2p}{100-2p}$. The demand function is unit elastic for $p = 25$, inelastic for $0 < p < 25$ and elastic for $25 < p < 50$.

b) $\varepsilon(p) = -\frac{1}{5p+1}$. The demand function is inelastic for all $p \geqslant 1$.

c) $\varepsilon(p) = -0.1p$. The demand function is unit elastic for $p = 10$, inelastic for $0 < p < 10$ and elastic for $p > 10$.

d) $\varepsilon(p) = -\frac{18}{p^2+9}$. The demand function is unit elastic for $p = 3$, elastic for $1 \leqslant p < 3$ and inelastic for $p > 3$.

e) $\varepsilon(p) = -0.04p^2$. The demand function is unit elastic for $p = 5$, inelastic for $0 < p < 5$ and elastic for $p > 5$.

Problem 3

a) $P_1(x) = 1 + x$, $P_2(x) = 1 + x + \frac{x^2}{2}$, $P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, $P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$

b) $P_1(1) = 2$, $P_2(1) = 2.5$, $P_3(1) = \frac{8}{3} \approx 2.67$, $P_4(x) = \frac{65}{24} \approx 2.71$. The distance from $f(1) = e$ equals (approximately):

$|f(1) - P_1(1)| = |e - 2| = 0.72$
$|f(1) - P_2(1)| = |e - 2.5| = 0.22$
$|f(1) - P_3(1)| = |e - \frac{8}{3}| = 0.052$
$|f(1) - P_4(1)| = |e - \frac{65}{24}| = 0.0099$

Problem 4

a) $P_1(x) = (x-1)$, $P_2(x) = (x-1) - \frac{(x-1)^2}{2}$, $P_3(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$,
$P_4(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$

b) $P_1(2) = 1$, $P_2(2) = \frac{1}{2}$, $P_3(1) = \frac{5}{6} \approx 0.83$, $P_4(x) = \frac{7}{12} \approx 0.58$. The distance from $f(2) = \ln(2)$ equals (approximately):

$|f(2) - P_1(2)| = |\ln(2) - 1| = 0.31$
$|f(2) - P_2(2)| = |\ln(2) - \frac{1}{2}| = 0.19$
$|f(2) - P_3(2)| = |\ln(2) - \frac{5}{6}| = 0.14$
$|f(2) - P_4(2)| = |\ln(2) - \frac{7}{12}| = 0.11$

Problem 5

a) $P_2(x) = 100 + (x-50) - 0.2(x-50)^2$

b) $f(52) \approx P_2(52) = 101.2$

Problem 6

a) This is a $\frac{0}{0}$-expression. Hence we can use l'Hôpital's rule. Differentiate numerator and denominator. Get another $\frac{0}{0}$-expression and use l'Hôpital's rule again:

$$\lim_{x\to 0} \frac{e^x - (1+x)}{x^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x - 1}{2x} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2} \quad \left(= \frac{f''(0)}{2} \right)$$

b)

$$\lim_{x\to 0} \frac{e^x - (1 + x + \frac{x^2}{2})}{x^3} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x - (1+x)}{3x^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x - 1}{6x} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x}{6} = \frac{1}{6} \quad \left(= \frac{f'''(0)}{3!} \right)$$

c) $\frac{1}{24}$ $\left(=\frac{f^{(4)}(0)}{4!}\right)$

Problem 7

a) This is a $\frac{0}{0}$-expression. Hence we can use l'Hôpital's rule. Differentiate numerator and denominator. Get another $\frac{0}{0}$-expression and use l'Hôpital's rule again:

$$\lim_{x\to 1}\frac{\ln(x)-(x-1)}{(x-1)^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 1}\frac{\frac{1}{x}-1}{2(x-1)} \overset{\text{l'Hôp}}{=} \lim_{x\to 1}\frac{-\frac{1}{x^2}}{2} = -\frac{1}{2} \quad \left(=\frac{f''(1)}{2}\right)$$

b)

$$\lim_{x\to 1}\frac{\ln(x)-[(x-1)-\frac{(x-1)^2}{2}]}{(x-1)^3} \overset{\text{l'Hôp}}{=} \lim_{x\to 1}\frac{\frac{1}{x}-[1-(x-1)]}{3(x-1)^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 1}\frac{-\frac{1}{x^2}+1}{6(x-1)} \overset{\text{l'Hôp}}{=} \lim_{x\to 1}\frac{\frac{2}{x^3}}{6} = \frac{1}{3} \quad \left(=\frac{f'''(1)}{3!}\right)$$

c) $-\frac{1}{4}$ $\left(=\frac{f^{(4)}(1)}{4!}\right)$

Problem 8 (Multiple choice exam 2017s, problem 12)
B

Problem 9 (Multiple choice exam 2016a, problem 12)
B