Key Problems

Problem 1.
Solve the systems of equations:

a) $\begin{aligned} 2x + 3y &= 14 \\ 7x - 4y &= 20 \end{aligned}$ b) $\begin{aligned} x^2 + y^2 &= 20 \\ x - y &= 2 \end{aligned}$ c) $\begin{aligned} x - 2y &= 6 \\ xy &= -4 \end{aligned}$ d) $\begin{aligned} x^2 - y^2 &= 8 \\ xy &= 3 \end{aligned}$

Problem 2.
Solve the equation $ax = b$ when

a) $a = b = 1$ b) $a = 1,\ b = 0$ c) $a = 0,\ b = 1$ d) $a = b = 0$

Problem 3.
Solve the systems of equations:

a) $$\begin{aligned} x + y + z &= 4 \\ x + 2y + 4z &= 9 \\ x + 3y + 9z &= 16 \end{aligned}$$

b) $$\begin{aligned} x - y + z &= 3 \\ 2x - 4y + z &= 1 \\ 3x - 5y + 2z &= 4 \end{aligned}$$

Problem 4.
Use Gaussian elimination to solve the linear systems:

$$a)\quad \begin{array}{rcrcrcr} x &+& y &+& z &=& 11 \\ x &+& 2y &+& 4z &=& 22 \\ x &-& y &+& z &=& 1 \end{array} \qquad b)\quad \begin{array}{rcrcrcr} x &+& y &+& z &=& 6 \\ x &+& 2y &+& 4z &=& 16 \\ x &+& 3y &+& 9z &=& 20 \end{array}$$

Problem 5.
Use Gaussian elimination to solve the linear systems. How many solutions are there?

$$a)\quad \begin{array}{rcrcr} x &+& 3y &=& 1 \\ x &-& y &=& 9 \\ 2x &+& 2y &=& 3 \end{array} \qquad b)\quad \begin{array}{rcrcr} x &+& 3y &=& 7 \\ x &-& y &=& 3 \\ 2x &+& 2y &=& 10 \end{array} \qquad c)\quad \begin{array}{rcrcrcr} x &+& y &+& z &=& 11 \\ x &-& y &+& z &=& 9 \\ 2x &+& 3y &+& 5z &=& 44 \\ 3x &-& y &+& 2z &=& 45 \end{array}$$

Problem 6.
Use Gaussian elimination to solve the linear systems. How many solutions are there?

$$a)\quad \begin{array}{rcrcrcr} x &+& 2y &+& 3z &=& 4 \\ -x &-& y &+& z &=& 1 \\ 3x &+& 4y &+& z &=& 2 \end{array} \qquad b)\quad \begin{array}{rcrcrcr} 3x &+& 4y &+& 3z &=& 2 \\ 2x &-& y &+& z &=& 1 \\ 7x &+& 2y &+& 5z &=& 3 \end{array}$$

Problem 7.
Use Gaussian elimination to solve the linear system. How many solutions are there?

$$\begin{array}{ccccccccc} x & + & y & + & z & + & w & = & 10 \\ x & + & 2y & + & 4z & - & w & = & 7 \\ x & - & y & + & z & + & 11w & = & 16 \end{array}$$

Problem 8.
A linear system is called *homogeneous* if all constant terms are zero. How many solutions does a homogeneous linear system with three equations and five variables have?

Problem 9.
Solve the system of equations:

$$2xy + y^3 + y^2 = 0$$
$$x^2 + 3xy^2 + 2xy = 0$$

Problem 10.
Optional: Problems from [Eriksen] (norwegian textbook)
Problem 6.1.1 - 6.1.6, 6.2.1 - 6.2.5, 6.3.1 - 6.3.7 (textbook)

Answers to Key Problems

Problem 1.

a) $(x,y) = (4,2)$

b) $(x,y) = (4,2), (-2,-4)$

c) $(x,y) = (2,-2), (4,-1)$

d) $(x,y = (3,1), (-3,-1)$

Problem 2.

a) $x = 1$ b) $x = 0$ c) no solutions d) all values of x are solutions

Problem 3.
a) $(x,y,z) = (1,2,1)$ *b)* $(x,y,z) = (-3z/2 + 11/2, -z/2 + 5/2, z)$ where z is a free variable

Problem 4.
a) $(x,y,z) = (4,5,2)$ b) $(x,y,z) = (-10,19,-3)$

Problem 5.
a) No solutions b) One solution $(x,y) = (4,1)$ c) No solutions

Problem 6.
a) Infinitely man solutions $(x,y,z) = (-6 + 5z, 5 - 4z, z)$ with z free b) No solutions

Problem 7.
Infinitely man solutions $(x,y,z) = (13 - 5w, -3 + 5w, -w, w)$ with w free

Problem 8.
Infinitely many solutions.

Problem 9.
Solutions: $(x,y) = (0,0), (0,-1), (3/25, -3/5)$