Key Problems

Problem 1.
Find the natural domain of definition D_f og range R_f of f:

a) $f(x,y) = 2x + 3y$ b) $f(x,y) = \sqrt{x + 3y}$ c) $f(x,y) = (2x - y)^{-3/2}$ d) $f(x,y) = 17x^{1.2}y^{3.4}$

Problem 2.
Find as many vectors as possible that are normal to the vector $\mathbf{v}$:

a) $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ b) $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ c) $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ d) $\mathbf{v} = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$

Problem 3.
Sketch the level curves $f(x,y) = c$ for the given values of c in the same coordinate system:

a) $f(x,y) = 2x + 3y$ og $c = -2, -1, 0, 1, 2$
b) $f(x,y) = x^2 + y^2$ og $c = -2, -1, 0, 1, 2$
c) $f(x,y) = 4x^2 + 9y^2$ og $c = -2, -1, 0, 1, 2$
d) $f(x,y) = x^2 - 2x + 4y^2$ og $c = -2, -1, 0, 1, 2$

Problem 4.
We consider the level curve $f(x,y) = c$ of the function $f(x,y) = x^2 + 4x + y^2 - 2y$. What kind of curve is this? Describe the gradient of f in a point on the level curve geometrically.

Problem 5.
Compute the partial derivatives f'_x og f'_y:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 - y$
c) $f(x,y) = 3x^2 + xy - y^2$
d) $f(x,y) = x^3 + 3xy + 2y^3 - 2x$
e) $f(x,y) = x^2 \ln y$
f) $f(x,y) = e^{xy}$
g) $f(x,y) = xe^y - ye^x$
h) $f(x,y) = \sqrt{x^2 + y^2}$
i) $f(x,y) = \ln(x^2 + xy + y^2)$

Problem 6.
Compute the partial derivatives f'_x og f'_y:

a) $f(x,y) = \dfrac{1}{x + y}$
b) $f(x,y) = \dfrac{2x + 3y}{xy}$
c) $f(x,y) = \dfrac{xy}{2x - y}$
d) $f(x,y) = \dfrac{1}{x^2 + y^2}$
e) $f(x,y) = \dfrac{1}{x} + \dfrac{1}{y}$
f) $f(x,y) = \dfrac{x}{y} - \dfrac{y}{x}$

Problem 7.
Describe the graph of $f(x,y) = 3x - 4y + 1$ geometrically.

Problem 8.
Optional: Problems from [Eriksen] (norwegian textbook)
Problem 7.1.1 - 7.1.4, 7.2.1 - 7.2.2, 7.3.1 - 7.3.2 (textbook)

Answers to Key Problems

Problem 1.

a) $D_f = \mathbb{R}^2,\ R_f = \mathbb{R}$

b) $D_f = \{(x,y) \in \mathbb{R}^2 : x + 3y \geq 0\},\ R_f = [0,\infty)$

c) $D_f = \{(x,y) \in \mathbb{R}^2 : 2x - y > 0\},\ R_f = (0,\infty)$

d) $D_f = \{(x,y) \in \mathbb{R}^2 : x,y \geq 0\},\ R_f = [0,\infty)$

Problem 2.
All linear combinations of the vectors:

a) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

b) $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

d) $\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} -7 \\ 4 \\ 0 \end{pmatrix}$

Problem 3.

a) Straight lines

b) Circles for $c > 0$

c) Ellipses for $c > 0$

d) Ellipses with center $(1,0)$ for $c > -1$

Problem 4.
The curve is a circle with center $(-2,1)$ and radius $\sqrt{c+5}$. The gradient points away from the center of the circle.

Problem 5.

a) $f'_x = 2,\ f_y = 3$

b) $f'_x = 2x,\ f_y = -1$

c) $f'_x = 6x + y,\ f_y = x - 2y$

d) $f'_x = 3x^2 + 3y - 2,\ f_y = 3x + 6y^2$

e) $f'_x = 2x \ln y,\ f_y = x^2/y$

f) $f'_x = ye^{xy},\ f_y = xe^{xy}$

g) $f'_x = e^y - ye^x,\ f_y = xe^y - e^x$

h) $f'_x = \dfrac{x}{\sqrt{x^2+y^2}}, \quad f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

i) $f'_x = \dfrac{2x+y}{x^2+xy+y^2}, \quad f_y = \dfrac{x+2y}{x^2+xy+y^2}$

Problem 6.

a) $f'_x = f'_y = -\dfrac{1}{(x+y)^2}$

b) $f'_x = -\dfrac{2}{y^2}, \quad f'_y = -\dfrac{3}{x^2}$

c) $f'_x = \dfrac{-y^2}{(2x-y)^2}, \quad f'_y = \dfrac{2x^2}{(2x-y)^2}$

d) $f'_x = \dfrac{-2x}{(x^2+y^2)^2},\ f'_y = \dfrac{-2y}{(x^2+y^2)^2}$

e) $f'_x = \dfrac{-1}{x^2}, \quad f'_y = \dfrac{-1}{y^2}$

f) $f'_x = \dfrac{1}{y} + \dfrac{y}{x^2}, \quad f'_y = \dfrac{-x}{y^2} - \dfrac{1}{x}$

Problem 7.
The graph of f is the plane the intersects the z-axis at $z = 1$ and has normal vector

$$\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$$