

**EBA2911 Mathematics for Business Analytics**  
**autumn 2020**  
**Exercises**

*... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.*

R. Lucas

**Lecture 7**

**on Wednesday 23 Sept. 10-11.45 in B2-060**  
**Sec. 4.1-6**

**Functions and graphs. Linear and quadratic functions. Revenue and cost functions.**

Here are recommended exercises from the textbook [SHSC].

- Section 4.2 exercise 1-6, 13-15
- Section 4.3 exercise 1-3
- Section 4.4 exercise 1-10
- Section 4.6 exercise 1-7

**Problems for the exercise session**  
**Wednesday 23 Sept. at 12-15 in CU1-067 or on Zoom**

**Problem 1** Determine the expression of the linear function  $f(x)$  in figure 1.

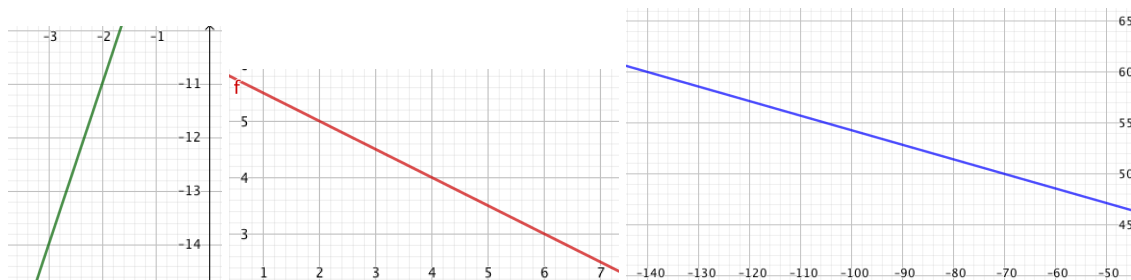


Figure 1: Linear functions (a-c)

**Problem 2** Determine the intersection points of the graph and the  $x$ -axis and of the graph and the  $y$ -axis in Problem 1 a-c.

**Problem 3** Determine the expression of the linear function  $f(x)$  such that the graph passes through the points  $P$  and  $Q$ .

- a)  $P = (-2, 5)$  and  $Q = (-4, 6)$
- b)  $P = (80, 90)$  and  $Q = (50, 80)$
- c)  $P = (4, -3)$  and  $Q = (-1, 7)$

**Problem 4** Determine the expression of the linear function  $f(x)$  such that the graph passes through the point  $P$  and has slope  $a$ .

- a)  $P = (-2, 5)$  and  $a = \frac{2}{3}$
- b)  $P = (8, 90)$  and  $a = \frac{1}{10}$
- c)  $P = (4, 30)$  and  $a = -\frac{3}{10}$

**Problem 5** Determine the expression of the second degree polynomial function  $f(x)$  in a-f, see figure 2 and 3.

**Problem 6** Determine the intersection points of the graph and the  $x$ -axis and of the graph and the  $y$ -axis in Problem 3 c-f.

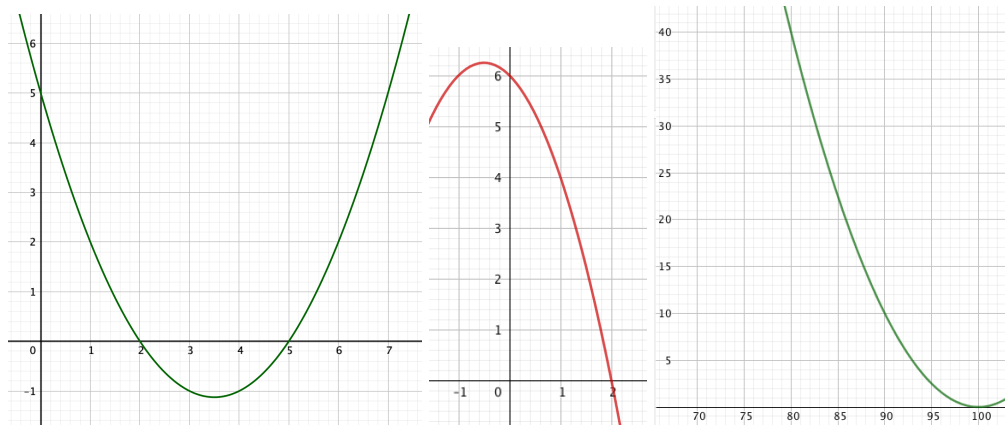


Figure 2: Parabolas (a-c)

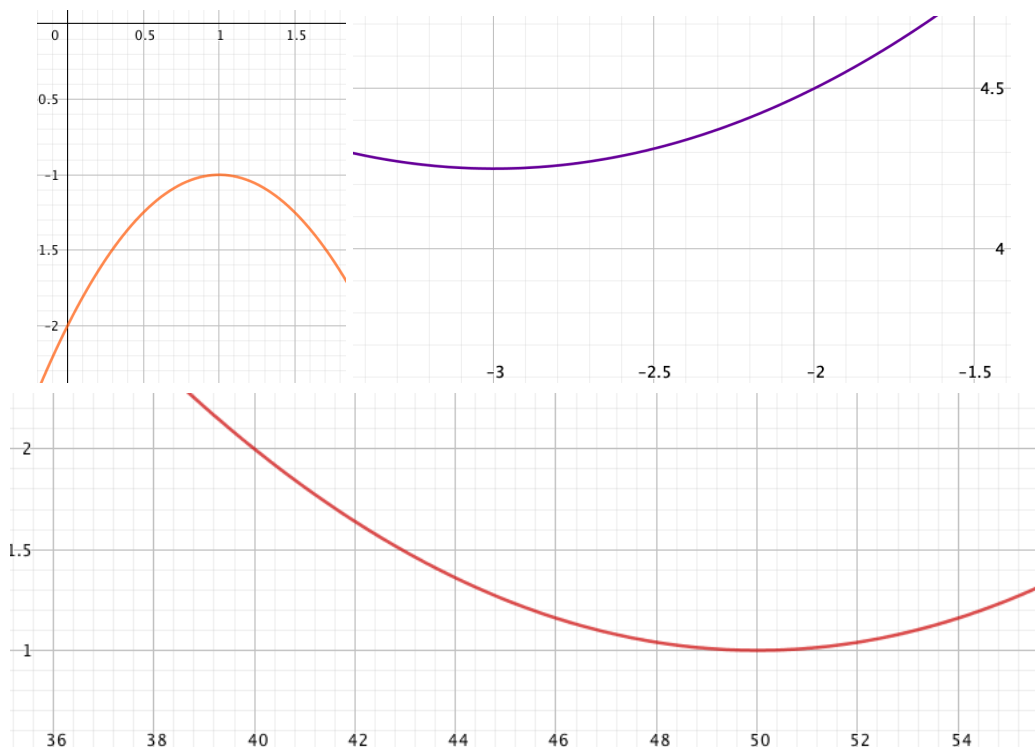


Figure 3: Parabolas (d-f)

**Problem 7** Determine the expression of the second degree polynomial function  $f(x)$  such that:

- a) The graph passes through the points  $P = (0, 7)$ ,  $Q = (1, 4)$  and  $R = (2, 3)$ .
- b) The graph passes through the points  $P = (-5, 65)$ ,  $Q = (3, 65)$  and  $R = (7, 17)$ .
- c) The graph passes through the point  $P = (4, -6)$  and  $Q = (\frac{13}{2}, -\frac{49}{4})$  is the point where the function attains its minimum.

**Problem 8** Write  $f(x)$  in the form  $a(x - s)^2 + d$  (by ‘completing the square’) and use the expression to sketch the graph.

- a)  $f(x) = x^2 - 10x + 30$
- b)  $f(x) = 3x^2 + 36x + 110$
- c)  $f(x) = -\frac{1}{7}x^2 + 2x - 6$

**Problem 9** Suppose  $x$  is the number of units produced and sold. Determine the value of  $p$  (the unit price) which gives positive profit for  $x > 300$  (and negative profit for  $x < 300$ ) if:

- a) The cost function is  $C(x) = 2100 + 5x$  and the revenue function is  $R(x) = px$ .
- b) The cost function is  $C(x) = 4500 - 5x + 0.01x^2$  and the revenue function is  $R(x) = px$  (both with  $0 \leq x \leq 1000$  as domain of definition).

## Answers

### Problem 1

- a)  $f(x) = 3x - 5$
- b)  $f(x) = -\frac{x}{2} + 6$
- c)  $f(x) = -\frac{x}{7} + 40$

### Problem 2

- a)  $x = \frac{5}{3}$  and  $y = -5$
- b)  $x = 12$  and  $y = 6$
- c)  $x = 280$  and  $y = 40$

### Problem 3

- a)  $f(x) = -\frac{1}{2}x + 4$
- b)  $f(x) = \frac{1}{3}x + \frac{190}{3}$
- c)  $f(x) = -2x + 5$

### Problem 4

- a)  $f(x) = \frac{2}{3}x + \frac{19}{3}$
- b)  $f(x) = \frac{1}{10}x + \frac{446}{5}$
- c)  $f(x) = -\frac{3}{10}x + \frac{156}{5}$

### Problem 5

- a)  $f(x) = \frac{1}{2}(x-2)(x-5) = \frac{1}{2}x^2 - \frac{7}{2}x + 5$
- b)  $f(x) = -(x+3)(x-2) = -x^2 - x + 6$
- c)  $f(x) = \frac{1}{10}(x-100)^2 = \frac{1}{10}x^2 - 20x + 1000$
- d)  $f(x) = -(x-1)^2 - 1 = -x^2 + 2x - 2$
- e)  $f(x) = \frac{1}{4}(x+3)^2 + \frac{17}{4} = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{13}{2}$
- f)  $f(x) = \frac{1}{100}(x-50)^2 + 1 = \frac{1}{100}x^2 - x + 26$

### Problem 6

- a)  $x = 2, x = 5$  and  $y = 5$
- b)  $x = -3, x = 2$  and  $y = 6$
- c)  $x = 100$  and  $y = 1000$
- d) non and  $y = -2$
- e) non and  $y = \frac{13}{2}$
- f) non and  $y = 26$

### Problem 7

- a)  $(x-2)^2 + 3 = x^2 - 4x + 7$
- b)  $-(x+1)^2 + 81 = -x^2 - 2x + 80$
- c)  $(x - \frac{13}{2})^2 - \frac{49}{4} = x^2 - 13x + 30$

### Problem 8

- a)  $f(x) = (x-5)^2 + 5$
- b)  $f(x) = 3(x+6)^2 + 2$
- c)  $f(x) = -\frac{1}{7}(x-7)^2 + 1$

For sketches of a-c see figure 4. A small table with relevant function values is expected.

### Problem 9

- a)  $p = 3600/300 = 12$
- b)  $p = 13$

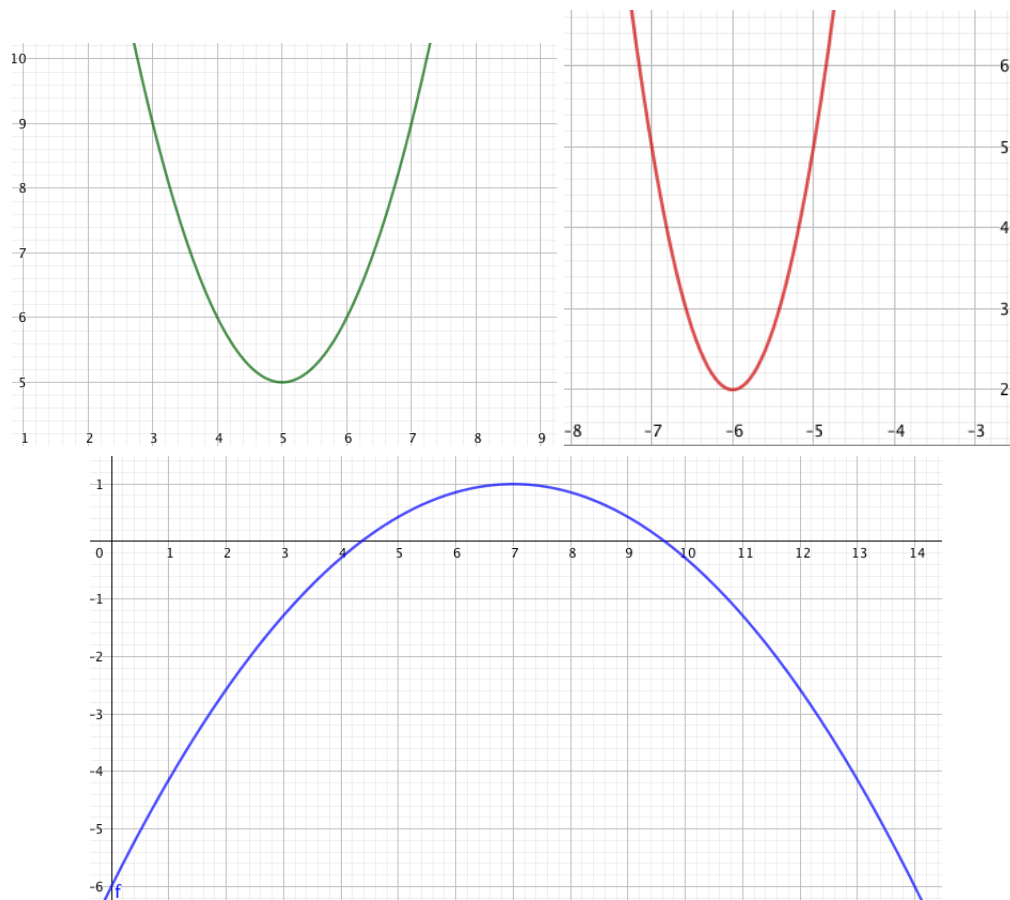


Figure 4: Parabolas in Problem 8 a-c