EBA2911, lecture 12, 1 Nov. 2021, Runar Ile

Plan 'Do' some problems from the Course Pages (EBA29101)

1. Financial math: Prob. 1 and 3c
2. Inverse functions and asymptotes: Prob 11 b
3. Parabolas: Prob 7
4. Inequalities and sign diag: Prob. 5c
5. ~~Hyperbolas: Prob. 10~~ no time

1. Financial mathematics

Prob 1 a i) $6000 \cdot 1.0025^{96} + 6000 \cdot 1.0025^{95} + \ldots + 6000 \cdot 1.0025^{25}$

is a geom. series which I read from right to left.

Then the first term $a_1 = 6000 \cdot 1.0025^{25}$

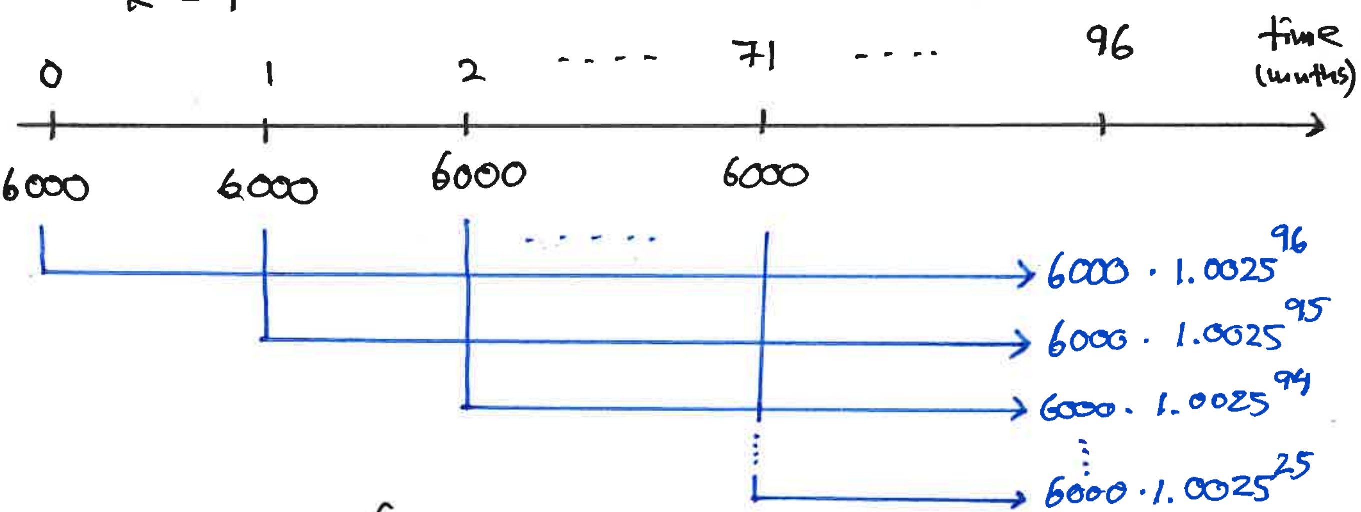
the multiplication factor $k = 1.0025$

the number of terms $n = 96 - 24 = 72$

By the formula for the sum of a geom. series the sum is

$$a_1 \cdot \frac{k^n - 1}{k - 1} = 6000 \cdot 1.0025^{25} \cdot \frac{1.0025^{72} - 1}{0.0025} = \underline{\underline{503122.08}}$$

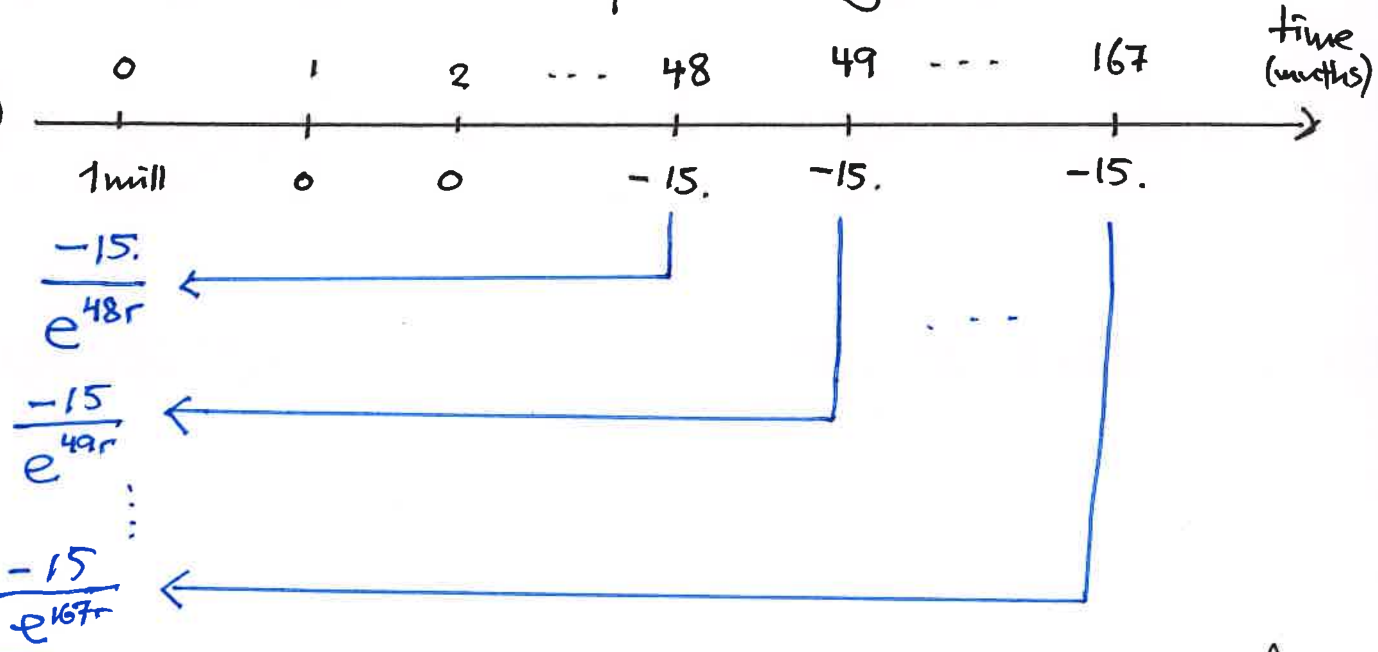
ii)



The sum is the future value at 96 of this cash flow.

E.g. Deposit 6000 each month for 6 years, starting today ($6 \cdot 12 = 72$ deposits)
The nominal interest is $0.025 \cdot 12 = 3\%$ with monthly compounding. The sum is the balance after 8 years.

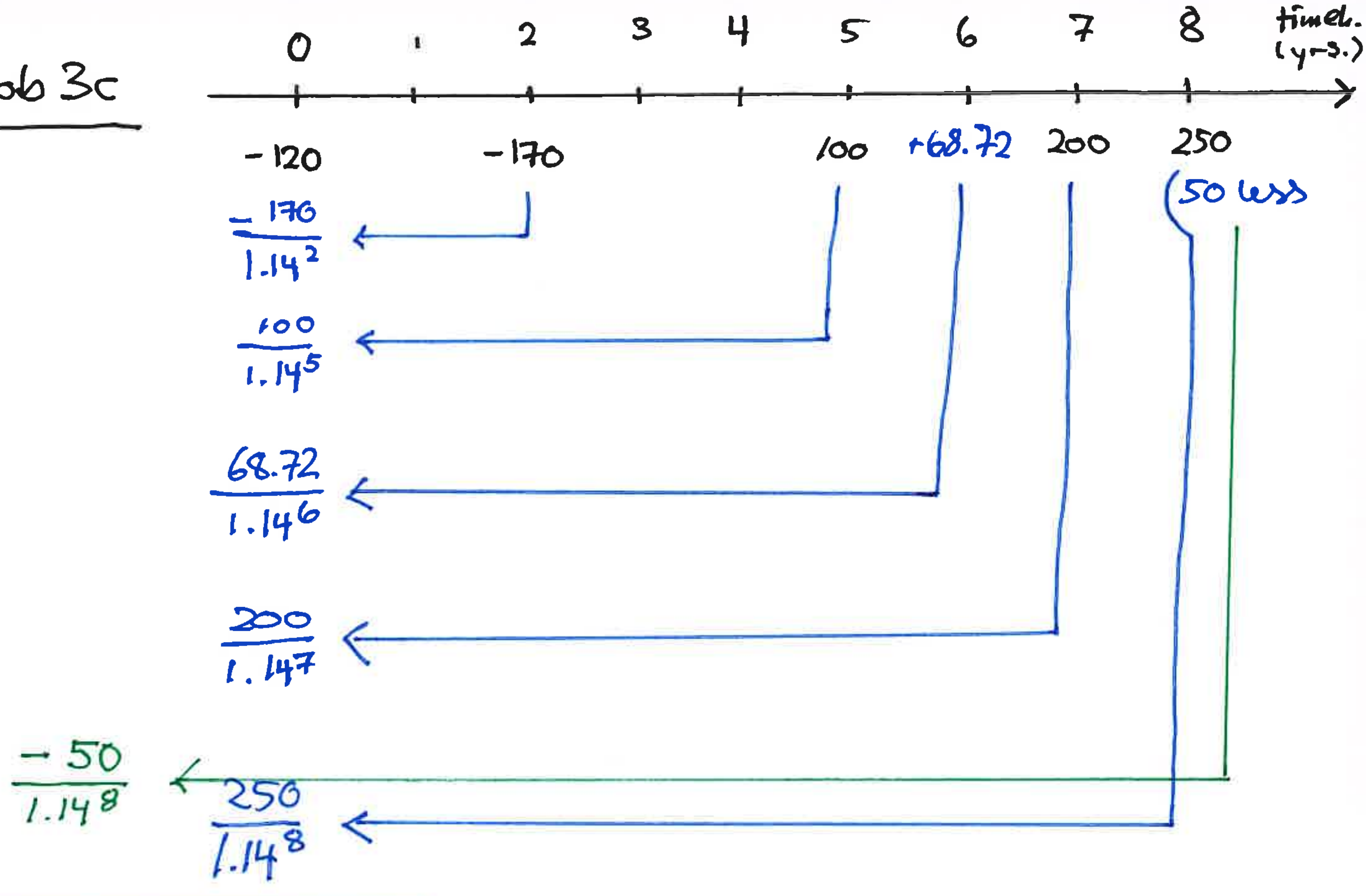
1b)



The sum = tot. present value of the cash flow with nominal period interest r and continuous compounding. The solution of the equation gives r as the IRR.

E.g. You borrow 1 mill. today and repay 15000 each month for 10 years ($167 - 47 = 120$ repayments)
First payment $4 = \frac{48}{12}$ years from now.
The solution gives the nominal period interest (for 1 month).

Prob 3c



The sum $= 0$

Need sum: $\frac{-50}{1.14^8} \xrightarrow{\cdot 1.14^2} \frac{-50}{1.14^6} = \underline{\underline{-22.78}}$ and $\begin{array}{r} 170 \\ -22.78 \\ \hline = 147.22 \end{array}$

$= -31.31$

Start: 16.01

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2. Inverse functions and asymptotes

Prob 11b $\quad f(x) = e^{-0.1x+2} + 5 \; , \quad D_f = [10, \infty\rangle$

① Solve the eq. $\quad y = e^{-0.1x+2} + 5 \quad$ for $x \quad | -5$

$$y - 5 = e^{-0.1x+2}$$

Insert both sides into $\ln(-)$.

$$\ln(y-5) = \ln\left(e^{-0.1x+2}\right) = -0.1x + 2 \qquad -2 \,|$$

$$-2 + \ln(y-5) = -0.1x \qquad | \cdot -10$$

$$20 - 10\ln(y-5) = x$$

② Change variables $x \leftrightarrow y$ and get $\underline{\underline{g(x) = 20 - 10 \cdot \ln(x-5)}}$

③ Always $R_g = D_f = \underline{\underline{[10, \rightarrow\rangle}}$

and $D_g = R_f$ — but what is R_f ?

Note that $f(x) = e^{-0.1x} \cdot e^2 + 5 = \dfrac{e^2}{e^{+0.1x}} + 5$

When x increases then $f(x)$ decreases (since $e^{0.1x}$ increases). So $f(x)$ is $\underbrace{\text{strictly}}_{z}$ decreasing

The max. value of $f(x)$ is then $f(10) = \dfrac{e^2}{e^1} + 5$

$= \underline{e + 5}$.

Moreover: $f(x)$ has a horizontal asymptote

$y = 5$ since $\dfrac{e^2}{e^{0.1x}} + 5 \xrightarrow[x \to \infty]{} 0^+ + 5 = 5^+$

So since $f(x)$ is decreasing and continuous $f(x)$ obtains all values between $e+5$ and 5 but is never equal to 5:

$\dfrac{e^2}{e^{0.1x}} + 5 = 5$ that is $\dfrac{e^2}{e^{0.1x}} = 0$

which has no solutions.

Conclusion: $D_g = R_f = \underline{\underline{\langle 5, 5+e]}}$

3. Parabolas

Prob 7 To find the zeros (roots) of $f(x)$ we first determine the std. form

$$f(x) = a(x-s)^2 + d$$ and then

solve eq. $f(x) = 0$ for x.

Here $x = s$ is (vertical) symmetry axis which seems to be $x = 200$

d gives the max. (min.) y-value which seems to be $y = 500$.

Then $f(x) = a(x-200)^2 + 500$

We also observe that $f(210) = 495$

that is $a \cdot (210-200)^2 + 500 = 495$

— " — $100a = 495 - 500$

$$a = \frac{-5}{100} = \underline{-0.05}$$

so $f(x) = -0.05(x-200)^2 + 500$

solve eq. $-0.05(x-200)^2 + 500 = 0$

that is $-0.05(x-200)^2 = -500 \quad | : -0.05$

$$(x-200)^2 = \frac{-500}{-0.05} = 10\,000 = 100^2$$

So either $x - 200 = 100$ or $x - 200 = -100$

so $\underline{\underline{x = 300 \text{ or } x = 100}}$

4. Inequalities with sign diag.

Prob 5c $\dfrac{\ln(x)+2}{e^x-4} \geq 0$

Note: Only defined for $x > 0$ because of $\ln(x)$.

To draw a sign diag. we need

(*) 0 on the right hand side - ok.

(*) the zeros of the numerator and the denominator

num: $\ln(x) + 2 = 0$

$\ln(x) = -2$

insert into $e^{(-)}$

$x = e^{\ln(x)} = \underline{e^{-2}}$

Since $\ln(x)+2$ is increasing it changes sign from $-$ to $+$ at $x = e^{-2}$

denom: $e^x - 4 = 0$

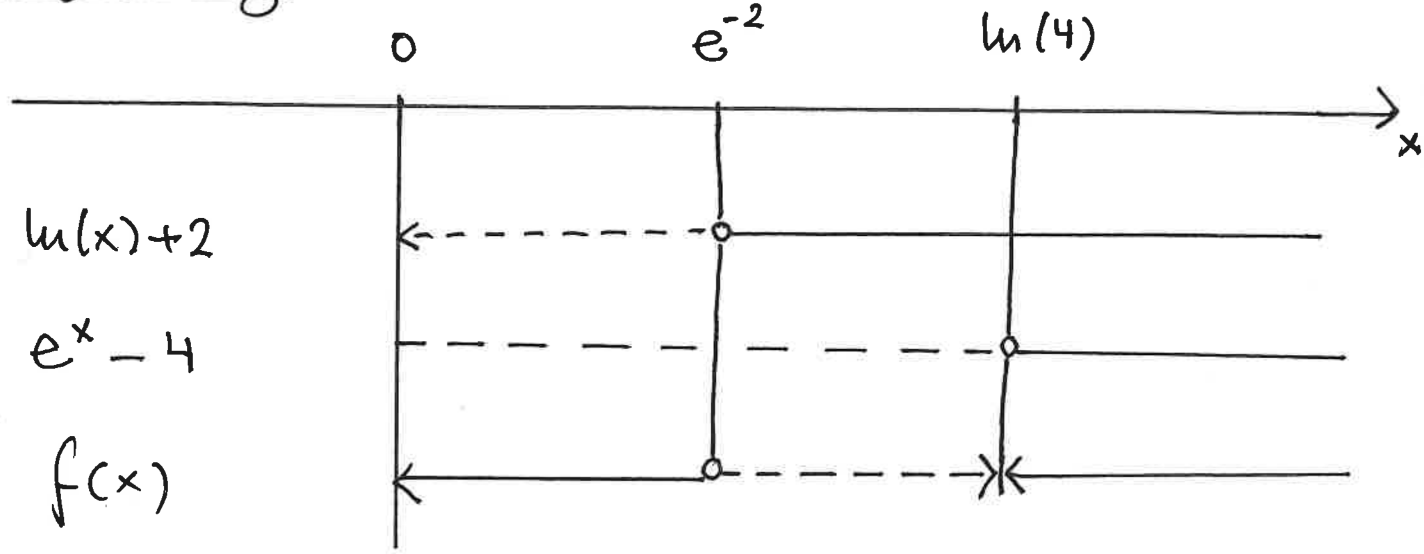
$e^x = 4$

$x = \ln(e^x) = \underline{\ln(4)}$

Since $e^x - 4$ is increasing it changes sign from $-$ to $+$ at $x = \ln(4)$

Also $0 < e^{-2} < 1 < \ln(4)$.

Sign diag



so $f(x) \geq 0$ for $\underline{\underline{0 < x \leq e^{-2} \text{ or } x > \ln(4)}}$