EBA 2911, lecture 15, 17 Nov. 2021, Runar Ile

Plan
1. l'Hôpital's rule
2. Marginal cost, average unit cost, marginal revenue

1. l'Hôpital's rule

Limits of the type $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$

Notation $\lim_{x \to 5} f(x)$ is the number which $f(x)$ is approaching when x is approaching 5.

Ex $f(x) = \dfrac{3x-3}{\ln(x)}$. Want to find $\lim_{x \to 1} f(x)$

$$\left.\begin{array}{lll} \text{Numerator} & 3x-3 \xrightarrow[x \to 1]{} & 3\cdot 1 - 3 = 0 \\ \text{Denominator} & \ln(x) \xrightarrow[x \to 1]{} & \ln(1) = 0 \end{array}\right\} \frac{0}{0}\text{-expression}$$

Then we can use l'Hôpital's rule:

$$\lim_{x \to 1} f(x) \overset{\text{l'Hôp.}}{=} \lim_{x \to 1} \frac{(3x-3)'}{[\ln(x)]'} = \lim_{x \to 1} \frac{3}{\frac{1}{x}} = \frac{3}{\frac{1}{1}} = \underline{\underline{3}}$$

Check: $x = 1.01$ then $\dfrac{3 \cdot 1.01 - 3}{\ln(1.01)} = 3.0150$

Note Has to be $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$!

Ex Use l'Hôpital's rule to determine the limit

$$\lim_{x\to 0} \frac{2x}{e^x - 1}$$

Solution Numerator: $2x \xrightarrow[x\to 0]{} 0$

Denominator: $e^x - 1 \xrightarrow[x\to 0]{} e^0 - 1 = 1 - 1 = 0$

so we have a $\frac{0}{0}$-situation, and can apply l'Hôpital's rule:

$(2x)' = 2$ and $(e^x - 1)' = e^x$ so

$$\lim_{x\to 0} \frac{2x}{e^x - 1} = \lim_{x\to 0} \frac{2}{e^x} = \frac{2}{e^0} = \frac{2}{1} = \underline{\underline{2}}$$

Ex $$\underbrace{\lim_{x\to\infty} \frac{x^2}{e^x}}_{\frac{\infty}{\infty}} \overset{\text{l'Hôp}}{=} \underbrace{\lim_{x\to\infty} \frac{2x}{e^x}}_{\frac{\infty}{\infty}} \overset{\text{l'Hôp}}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0$$

---

2. Marginal cost, average unit cost, marginal revenue

$C(x)$ is the total cost of producing x units of some commodity.

$C'(x)$ is the marginal cost

Interpretation: The cost of producing one more unit than x units.

$$= C(x+1) - C(x) = \frac{C(x+1) - C(x)}{1} \approx \lim_{h\to 0} \frac{C(x+h) - C(x)}{h} = C'(x)$$

Why $C'(x)$? – much simpler math. to work with!

$R(x)$ is the <u>revenue</u> by selling x units.

$R'(x)$ is the marginal revenue function

<u>Ex</u> x = tons of sold salmon

$R'(50)$ = extra revenue by selling 1 extra ton of salmon more than 50 tons.

(so $\approx R(51) - R(50)$)

<u>The profit function</u> (x = produced and sold units)

$P(x) = R(x) - C(x)$ (economists: $\Pi(x)$)

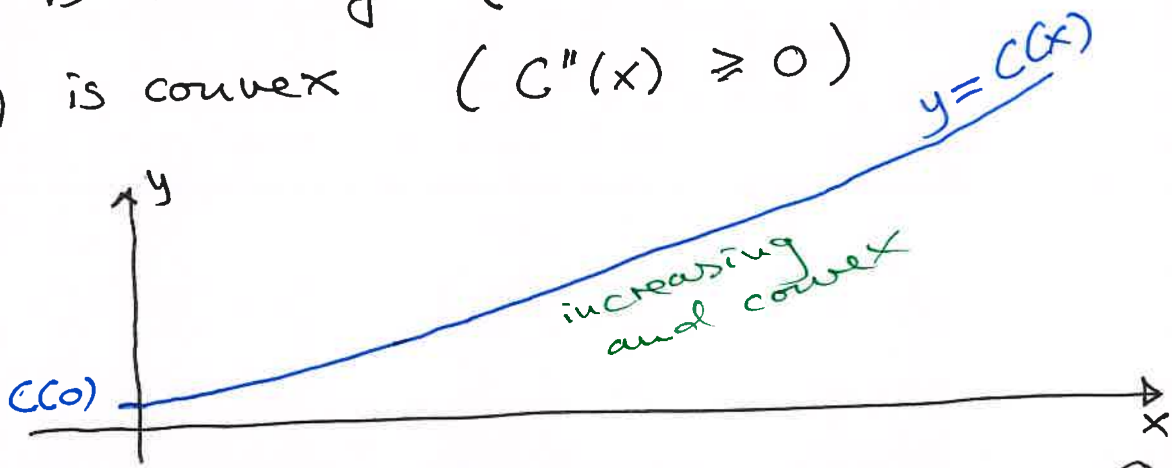
$P'(x)$ is the marginal profit function.

<u>Average unit cost of producing</u> x units is $A(x) = \dfrac{C(x)}{x}$

Start 10.55

<u>Definition</u> $C(x)$ is a cost function if

1. $C(0) > 0$ (start-up cost)
2. $C(x)$ is increasing ($C'(x) \geq 0$)
3. $C(x)$ is convex ($C''(x) \geq 0$)



Definition If $x = c$ is a minimum point for $A(x)$ then c is called the cost optimum (the x-value that gives the minimal average unit cost)

Result If $C''(x) > 0$ for all $x > 0$, then the cost optimum is the solution of the equation

$$C'(x) = A(x)$$

Ex $C(x) = x^2 + 200x + 160\,000$

This is a cost function because:

① $C(0) = 160\,000 > 0$

② $C'(x) = 2x + 200 > 0$ for $x \geq 0$

③ $C''(x) = 2 > 0$ (for all x!)

Then the cost optimum is (by the result) the solution of the equation

$$C'(x) = A(x)$$

$$2x + 200 = \frac{x^2 + 200x + 160\,000}{x}$$

$$\cancel{2x} + \cancel{200} = \cancel{x} + \cancel{200} + \frac{160\,000}{x}$$

$$x = \frac{160\,000}{x} \quad \Big| \cdot x$$

$$x^2 = 160\,000$$

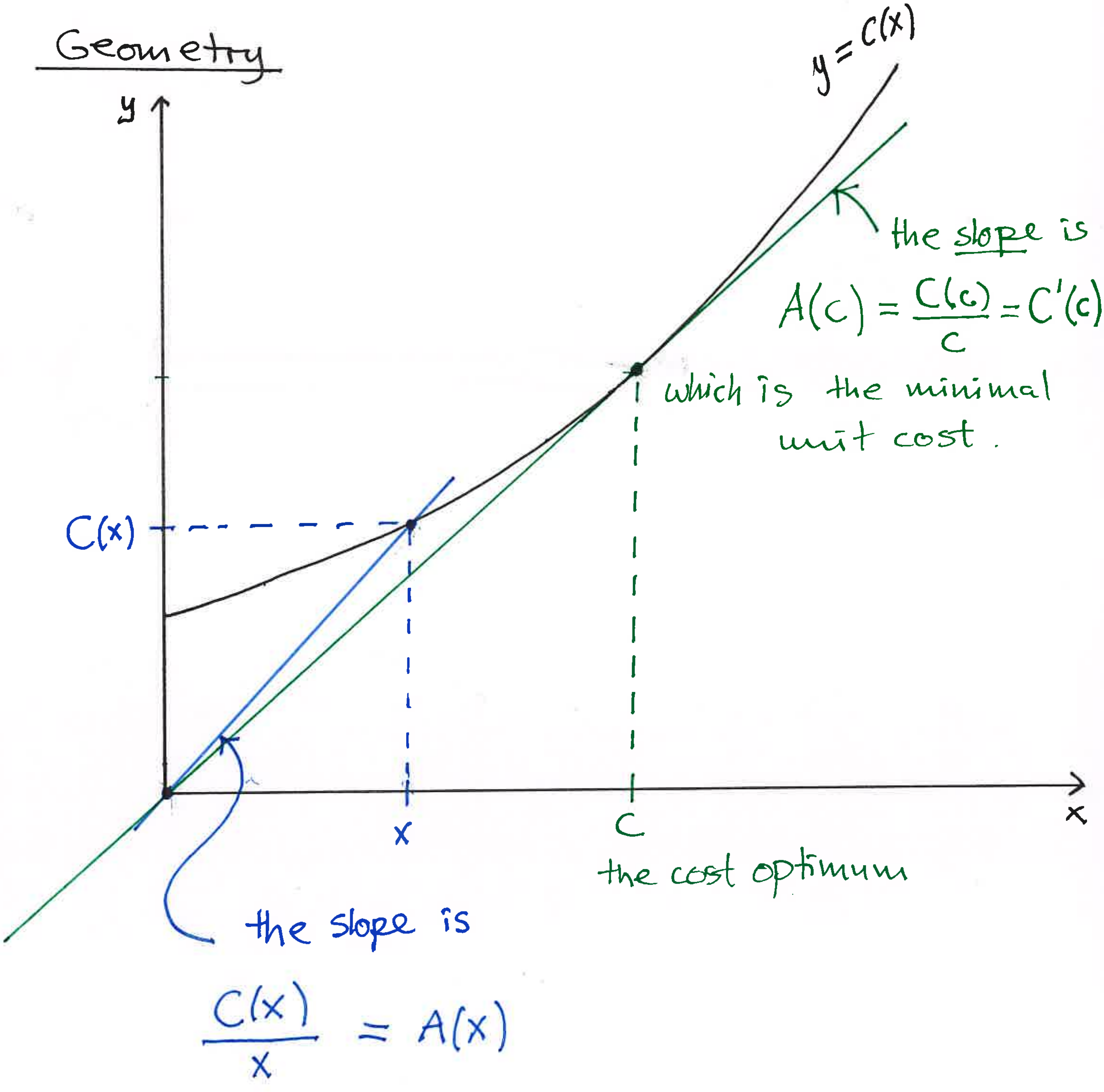
So $\underline{x = 400}$ (only pos. x)

is the cost optimum.

The minimal average unit cost is

$$A(400) = C'(400) = 2\cdot 400 + 200 = \underline{1000}$$

Geometry



and $A(c) = \frac{C(c)}{c}$ is the minimal average unit cost when $C'(c) = A(c)$.

– the smallest slope!