

Plan

- 1 Constrained optimization
- 2 Extreme value theorem

No working mic's
in this room, sorry.

Course paper: Solutions are posted.

① Constrained optimization

max/min $f(x,y) = x^2 + y^2$ when $0 \leq x, y \leq 1$
 objective fn. constraints

Constrained optimization

$D = \{(x,y) : 0 \leq x, y \leq 1\}$ in \mathbb{R}^2
 set of admissible pts.

max/min $f(x,y) = x^2 + y^2$

Unconstrained optimization

Unconstrained $\max/\min f(x,y)$

Constrained $\max/\min f(x,y)$ when (x,y) is in D

Candidate pts:

Candidate pts:

i) Stat. pts. $\therefore f'_x = f'_y = 0$

i) interior stat. pts: $f'_x = f'_y = 0$

ii) Other critical pts: f'_x or f'_y not def.

ii) interior critical pts: f'_x or f'_y does not exist

iii) boundary pts of D

iii) Boundary pts of D , where $D = \{(x,y) : \text{satisfy all constraints}\}$

(local) classification:

Second derivative test
 \rightarrow local max, local min, saddle pt.

EVT: if D is compact (closed + bounded) then there is a max/min.

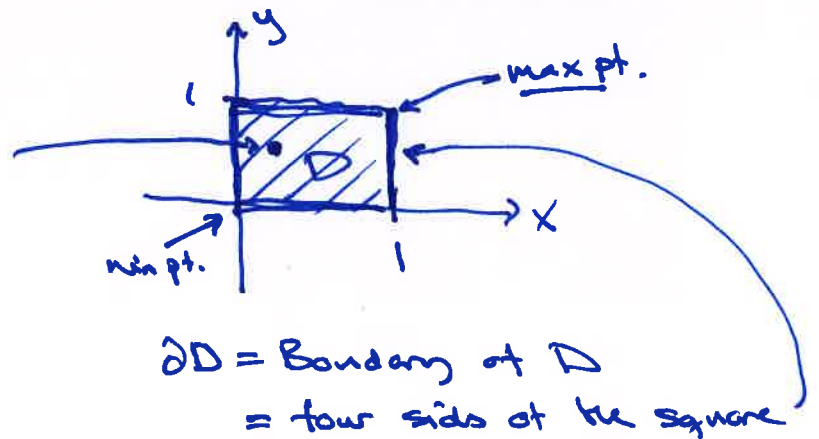
are any of these pts (global) max/min

Determine whether candidate pts are (global) max/min: Use EVT if D is compact.

Ex: max/min $f(x,y) = x^2 + y^2$ where $0 \leq x, y \leq 1$

z

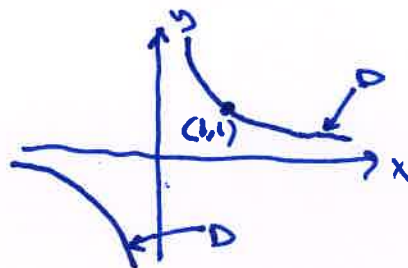
interior
pt. of D



Ex:

min $f(x,y) = x^2 + y^2$ where $xy = 1$

no interior pts.



$$xy = 1 \Rightarrow y = 1/x$$

$\partial D = \text{Boundary pts of } D = D$
(all pts. on D)

Ex: $\max/\min f(x,y) = x^2 + y^2$ wh $-1 \leq x, y \leq 1$

Candidate pts:

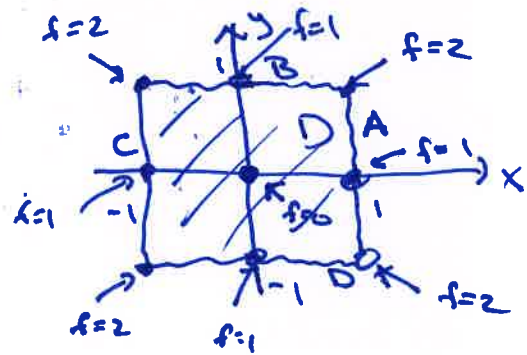
i) $f'_x = 2x = 0 \Rightarrow x = 0$
 $f'_y = 2y = 0 \Rightarrow y = 0$

$(0,0)$ interior pt of $D \Rightarrow$

Candidate i): $(x,y) = (0,0)$
 $f(0,0) = 0$

ii) other critical pts in the interior of D : none

iii) Boundary pts: $\partial D =$ four sides



A: $x=1, -1 \leq y \leq 1$

B: $y=1, -1 \leq x \leq 1$

C: $x=-1, -1 \leq y \leq 1$

D: $y=-1, -1 \leq x \leq 1$

Evi: f cont: \checkmark (ok)

D compact: \checkmark (ok)

closed \checkmark
 bounded \checkmark

there is a max/min
 \Rightarrow there is a max and a min among the candidate pts.

Candidate pts: compare values

i) $(0,0)$: $f(0,0) = 0$

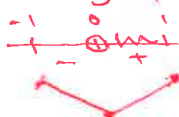
ii) none

iii)

A:	$f(1,y) = 1 + y^2, -1 \leq y \leq 1$ $\max: f(1,1) = f(1,-1) = 2$ $\min: f(1,0) = 1$
B:	$f(x,1) = x^2 + 1, -1 \leq x \leq 1$ $\max: f(1,1) = 2, f(-1,1) = 2$ $\min: f(0,1) = 1$
C:	$f(-1,y) = 1 + y^2, -1 \leq y \leq 1$ (similar with A) $\max: f(-1,1) = f(-1,-1) = 2$ $\min: f(-1,0) = 1$
D:	$f(x,-1) = x^2 + 1, -1 \leq x \leq 1$ (similar with B) $\max: f(1,-1) = f(-1,-1) = 2$ $\min: f(0,-1) = 1$

Alt: Side A
 $x=1$

$f(1,y) = 1 + y^2, -1 \leq y \leq 1$
 $(1+y^2)' = 2y$



$f(1,-1) = 2$ $f(1,1) = 2 \rightarrow \max$ on A
 $f(1,0) = 1 \rightarrow \min$ on A

Conclusion: D is compact, so there is max/min.

Highest value among cond. pts: $f_{\max} = \underline{\underline{2}}$
(at the max. pts. $(1,1), (-1,1), (1,-1), (-1,-1)$)

Lowest value — || — : $f_{\min} = \underline{\underline{0}}$
(at the min. pts $(0,0)$)

② Extreme value theorem:

$f(x,y)$ is a continuous fn. on a set D in \mathbb{R}^2

Extreme Value Theorem: (EVT)

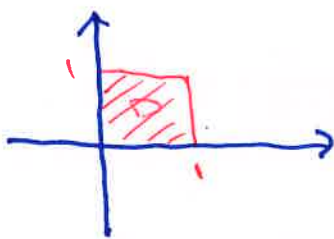
If f is a continuous fn. on a compact set D in \mathbb{R}^2 , then f has a maximum and a minimum on D .

Defn.: A subset D of \mathbb{R}^2 is called compact if it is closed and bounded.

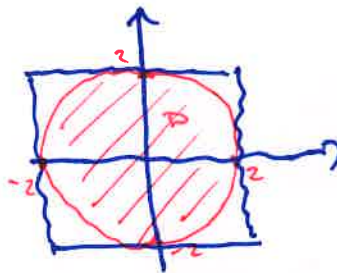
A subset D of \mathbb{R}^2 is closed if all boundary pts of D are included in D .

$= \leq \geq = \text{yes}$ $< > = \text{no}$

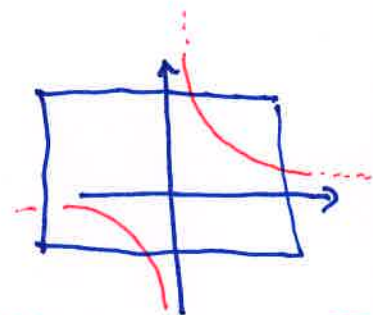
A subset D of \mathbb{R}^2 is bounded if there is a rectangle in \mathbb{R}^2 (with finite sides) that includes all of D



$D: 0 \leq x, y \leq 1$ (yes)



$D: x^2 + y^2 \leq 4$ (yes)



$D: xy=1$ (no)