EBA 2911, lecture 3, 6 Sept. 2021, Runar Ile

Plan: 1. Total present value of cash flow
2. Series
3. Annuities

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1. Total present value of cash flow.

Present value of an amount (K) paid n years (or periods) from now with interest r.
= what you have to deposit today (K_0) for the balance to become K n years from now if the interest is r

Since $K = K_0 \cdot (1+r)^n$

so that $K_0 = \dfrac{K}{(1+r)^n}$

Ex 50000 (K) 3 years from now with 4% interest has present value

$$K_0 = \frac{50\,000}{1.04^3} = \underline{\underline{44\,449.82}}$$

That is: If you deposit 44 449.82 today into an account earning 4% interest, the balance after 3 years will be 50 000.

Ex An investment of 120 mill. is supposed to give payments of 70 mill. 4 years from now and 90 mill. 6 years from now.

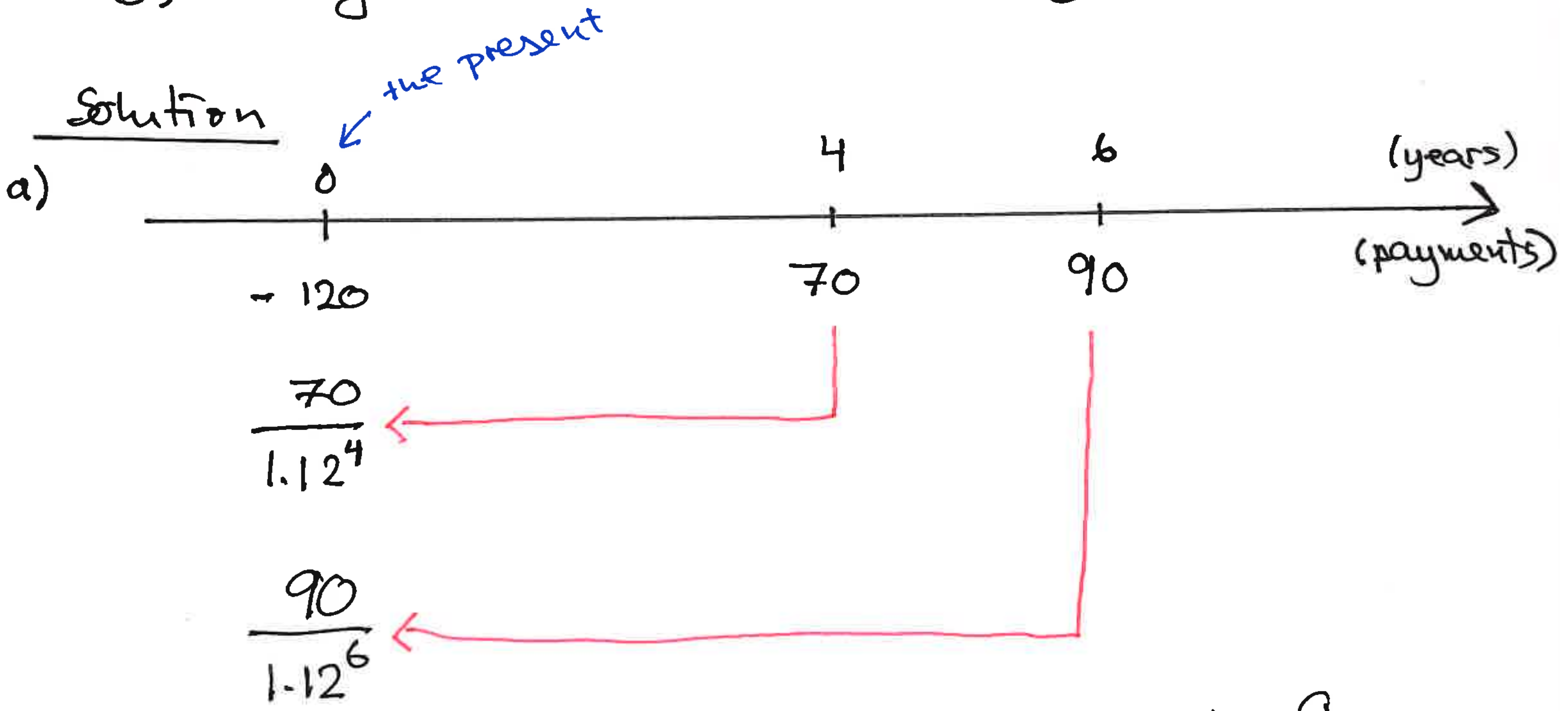
Suppose the interest is 12%.

a) Determine the the total present value of the cash flow.

b) Do you think this is a good investment?

Solution

a)



= total present value of the cash flow

$$= -120 + \frac{70}{1.12^4} + \frac{90}{1.12^6} = \underline{\underline{-29.92}}$$

b) We don't 12% interest on this investment

In fact (trying, plotting) the internal rate of return (the interest that makes the tot. pres. value of the cash flow equal to 0) is approx. 5.81% because

$$-120 + \frac{70}{1.0581^4} + \frac{90}{1.0581^6} = 0.0$$

5.81 % can be interpreted as the annual yield of the investment.

2. Series

– many terms added

third term

Ex $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{100}$ is a series with 10 terms.

third term

We write $a_1 + a_2 + a_3 + \ldots + a_{10}$

Geometric series $a_1 + a_2 + \ldots + a_n$

where each term is k times the previous term. (k is a number)

$$a_2 = k \cdot a_1$$
$$a_3 = k \cdot a_2 = k \cdot k \cdot a_1 = k^2 \cdot a_1$$
$$a_4 = k \cdot a_3 = k \cdot k^2 \cdot a_1 = k^3 \cdot a_1$$
$$\vdots$$
$$a_{10} = k^9 \cdot a_1$$

We can find a short expression for this series:

$$a_1 + a_2 + \ldots + a_n = a_1 + k \cdot a_1 + k^2 a_1 + k^3 a_1 + \ldots + k^{n-1} \cdot a_1$$
$$= a_1 \cdot \underbrace{(1 + k + k^2 + k^3 + \ldots + k^{n-1})}_{\frac{k^n - 1}{k - 1}}$$
$$= a_1 \cdot \frac{k^n - 1}{k - 1}$$

Problem Compute the sum

$$5 + 5\cdot 1.003 + 5\cdot 1.003^2 + 5\cdot 1.003^3 + \dots + 5\cdot 1.003^{60}$$

Solution This is a geometric series with $a_1 = 5$, $k = 1.003$ and $n = 61$

so the sum is $5\cdot\dfrac{1.003^{61} - 1}{1.003 - 1} = 5\cdot\dfrac{1.003^{61} - 1}{0.003}$

$\boxed{\text{Start: } 16.05}$ $= \underline{\underline{334.14}}$

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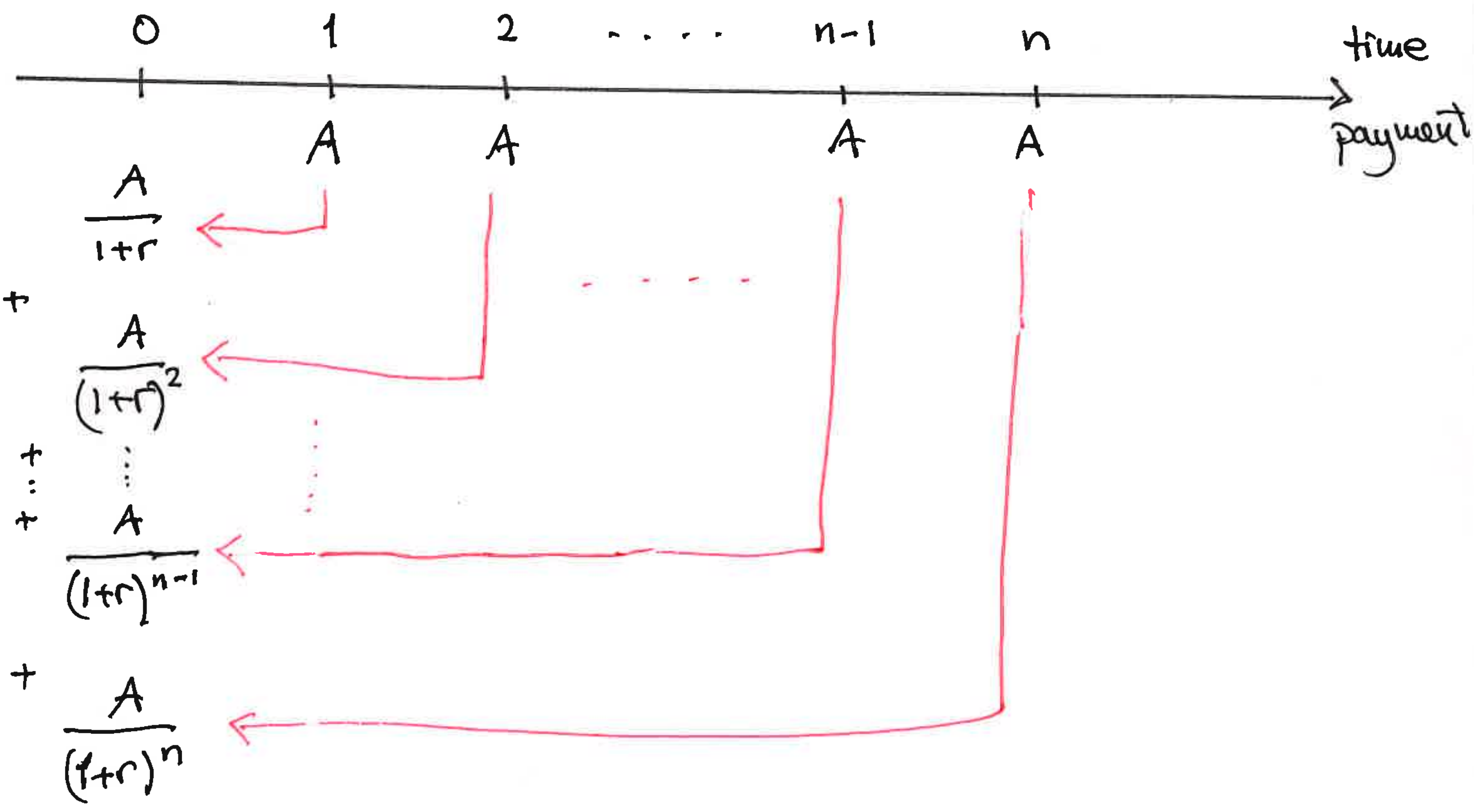
4. Annuities – regular cash flows



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the sum is the total present value of the regular cash flow. It is a geometric series with $a_1 = \frac{A}{1+r}$, the number of terms $= n$ and $k = \frac{1}{1+r}$

The sum is then (by the formula) $\dfrac{A}{1+r}\cdot\dfrac{\left(\frac{1}{1+r}\right)^n - 1}{\left(\frac{1}{1+r} - 1\right)}$ not so nice!

A finite geometric series is also a geometric series in the opposite direction!

Then $a_1 = \dfrac{A}{(1+r)^n}$ and $k = 1+r$ so

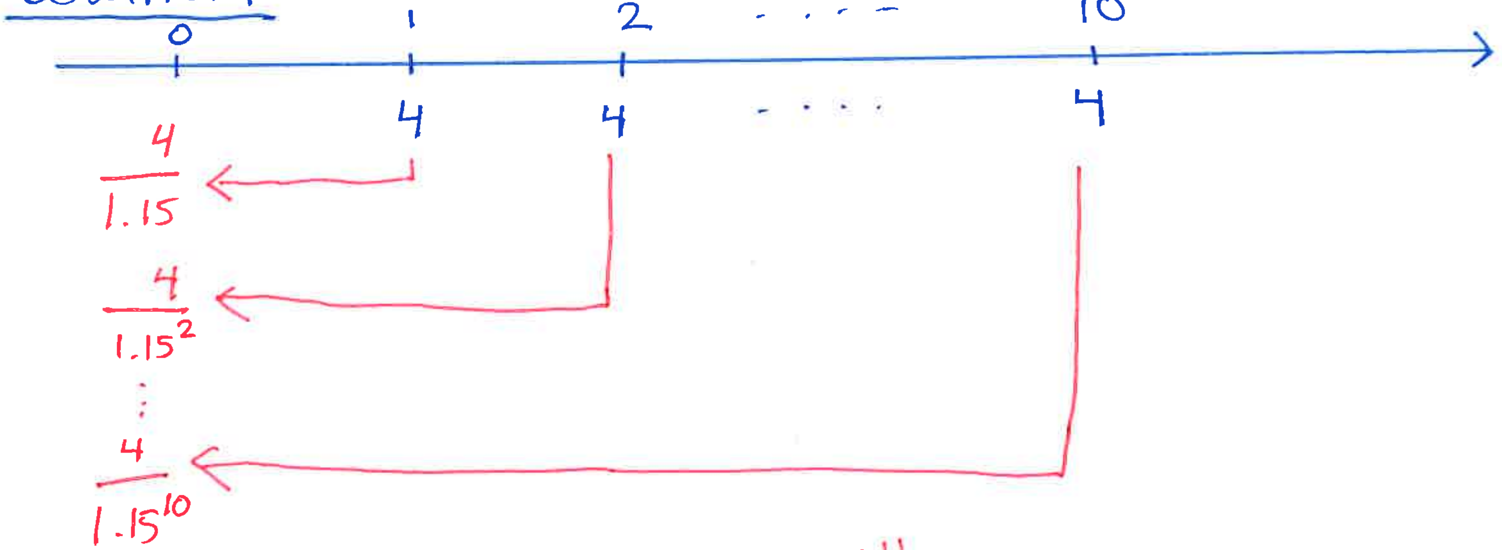
the sum is also

$$\frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{1+r-1} = \frac{A}{(1+r)^n} \cdot \frac{(1+r)^n - 1}{r}$$

— better!

<u>Problem</u> Hege considers an investment where 4 mill is paid every year for 10 years. The first payment is one year from now. Suppose the discount rate is 15 %. What is a fair price for this cash flow?

<u>Solution</u> We determine the tot. pres. val. of the cash flow.



the sum is a geom. series with

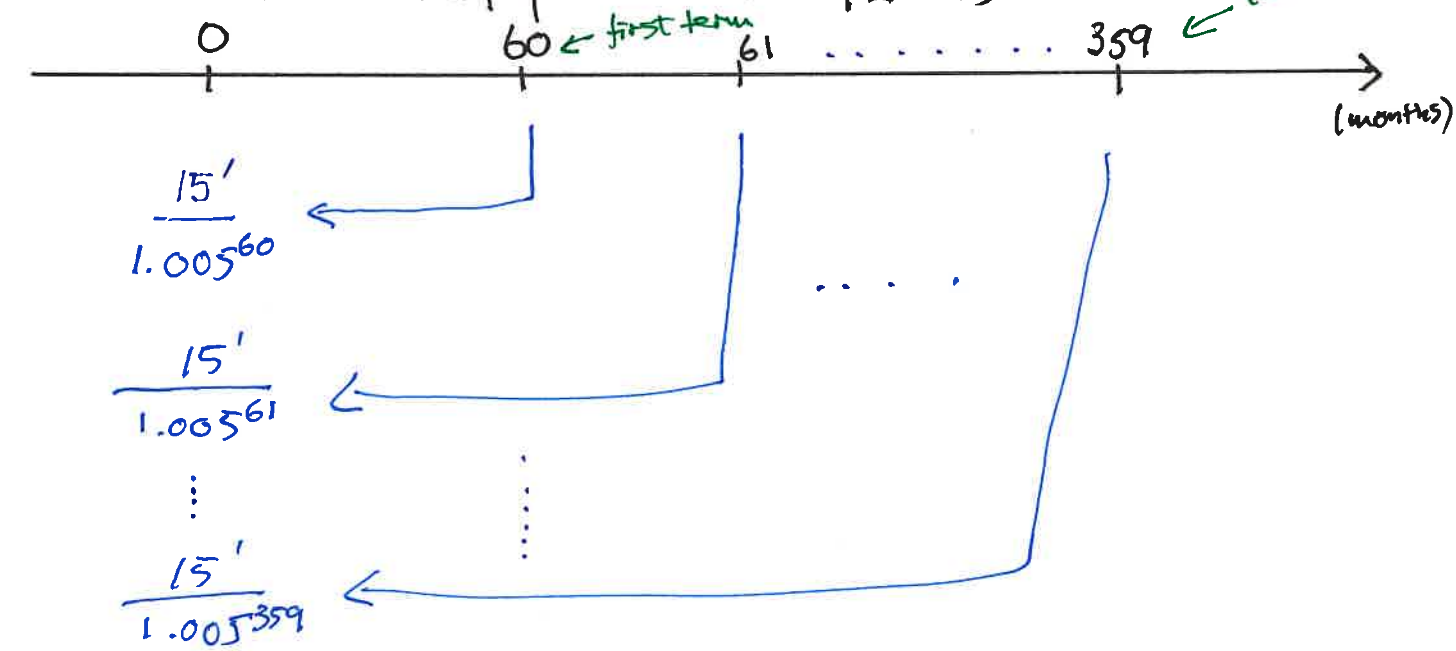
$a_1 = \dfrac{4}{1.15^{10}}$, $k = 1.15$, $n = 10$ so $\dfrac{4}{1.15^{10}} \cdot \dfrac{1.15^{10} - 1}{0.15} = \underline{\underline{20.08}}$

Ex (Term paper 2019a, probl. 6a)

Kåre considers a mortgage with monthly payments running for 25 years. The first payment is 5 years from now. Kåre reckons he can pay 15000 every month. The interest is 6%. Determine the geometric series which gives the tot. pres. val. of the cash flow. Calc. how much Kåre can borrow.

Solution The period rate is $\frac{6\%}{12} = 0{,}5\%$

Number of periods is $12 \cdot 25 = 300$



The sum is a geom. ser. with

$a_1 = \frac{15'}{1.005^{359}}$, $n = 300$, $k = 1.005$

The tot. pres. val.: $\frac{15000}{1.005^{359}} \cdot \frac{1.005^{300} - 1}{0.005} = \underline{\underline{1734620.76}}$

(what Kåre can borrow)