EBA 2911, lecture 5, 15 Sept. 2021, Runar Ile

1. Linear and quadratic equations
2. Equations with parametres: the abc-formula
3. Completing the square
4. Equations with given solutions

1. Linear and quadratic equations

A linear expression $ax + b$ (a and b are numbers and $a \neq 0$)

Ex $4x - 3$ $(a = 4,\ b = -3)$

A linear equation An equation which can be transformed into an equivalent equation of the form $ax + b = 0$ $(a \neq 0)$

Ex The eq. $\frac{1}{x+3} = \frac{2}{x+4}$ $\big|\cdot(x+3)(x+4)$

Multiply with a common denominator on each side.

is transformed to: $x + 4 = 2(x+3)$

use distributive law: $x + 4 = 2x + 6$

subtract $2x + 6$ on each side: $-x - 2 = 0$ $(a = -1,\ b = -2)$

$(x \neq -3,\ x = -4)$

A quadratic expression: $ax^2 + bx + c$

a, b, c are fixed numbers and $a \neq 0$.

A quad. eq: - an eq. which can be transformed into an equiv. eq $ax^2 + bx + c = 0$

Ex $\quad 3x+9 = (x-1)(x+3)$

* resolve parantheses and collect terms

$$3x+9 = x^2+3x-x-3$$

* subtract $3x+3$ on each side

$$x^2-x-12=0 \qquad (a=1,\ b=-1,\ c=-12)$$

Ex $\quad \frac{1}{x}+\frac{2}{x+1}=3 \qquad \Big|\cdot x(x+1)$

$$x+1+2x = 3x(x+1)$$

$$3x+1 = 3x^2+3x$$

subtr. $3x+1$ on each side

$(x\neq 0,\ x\neq -1)\qquad 3x^2-1=0 \qquad (a=3,\ b=0,\ c=-1)$

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2. Equations with parametres: the abc-formula

If $a\neq 0$ the solutions to any quadratic eq. on standard form $ax^2+bx+c=0$ are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- is called the quadratic formula (abc-formula)

Ex $3x^2 + 4x - 5 = 0$ $(a=3,\ b=4,\ c=-5)$

The quad. formula gives

$$x = \frac{-4 \pm \sqrt{4^2 - 4\cdot 3\cdot(-5)}}{2\cdot 3}$$

$$= \frac{-4 \pm \sqrt{16+60}}{6} = \frac{-4 \pm \sqrt{76}}{6}$$

$$= \frac{-4 \pm \sqrt{4\cdot 19}}{6} = \frac{-4 \pm \sqrt{4}\cdot\sqrt{19}}{6}$$

$$= \frac{\overset{2}{\cancel{-4}} \pm \overset{1}{\cancel{2}}\sqrt{19}}{\underset{3}{\cancel{6}}} = \frac{-2 \pm \sqrt{19}}{3}$$

$$= -\frac{2}{3} \pm \frac{\sqrt{19}}{3}$$

Three cases: $b^2 - 4ac > 0$ gives two solutions

$b^2 - 4ac = 0$ gives one solution

$b^2 - 4ac < 0$ gives no solutions.

Problem Determine the number of solutions:

a) $x^2 + 5x + 6 = 0$ $5^2 - 4\cdot 1\cdot 6 = 25 - 24 = 1 > 0$: two

b) $-x^2 + 2x - 1 = 0$ $2^2 - 4\cdot(-1)\cdot(-1) = 0$: one solut.

c) $4x^2 - 5x - 5 = 0$ $(-5)^2 - 4\cdot 4\cdot(-5) > 0$: two sol.

Start: 11.00

The quad. formula is often inefficient:

Ex $-3x^2 + 7 = 0$ $(a = -3,\ b = 0,\ c = 7)$

$$-3x^2 = -7 \quad | : (-3)$$

$$x^2 = \frac{-7}{-3} = \frac{7}{3}$$

$$|x| = \sqrt{x^2} = \sqrt{\frac{7}{3}}$$

so $\underline{\underline{x = \pm\sqrt{\frac{7}{3}}}}$

Ex $2x^2 - 6x = 0$ $(a = 2,\ b = -6,\ c = 0)$

$$2(x^2 - 3x) = 0 \quad | : 2$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

Either $\underline{\underline{x = 0}}$ or $x - 3 = 0$

$\underline{\underline{x = 3}}$

Pattern If $a \cdot b = 0$ then $a = 0$ or $b = 0$

(or both)

3. Completing the square

Ex $\underbrace{x^2 + 6x}_{} - 16 = 0$

Claim: $x^2 + 6x = (x+3)^2 - 9$

- because $(x+3)^2 = x^2 + 2\cdot 3\cdot x + 3^2$

$6 : 2$

$= \underbrace{x^2 + 6x}_{} + 9$

$$\underbrace{(x+3)^2 - \overset{3^2}{\overset{\shortparallel}{9}}}_{} - 16 = 0$$

$$(x+3)^2 = 25$$

so $x + 3 = 5$ or $x + 3 = -5$

$\underline{\underline{x = 2}}$ or $\underline{\underline{x = -8}}$

Problem Solve the quad. eq by completing the square.

a) $\underbrace{x^2 - 8x}_{} - 33 = 0$

b) $x^2 + 2x = 63$

Solutions a) $\frac{-8}{2} = -4$ so $x^2 - 8x = (x-4)^2 - 4^2$

(because $(x-4)^2 = x^2 - 2\cdot 4\cdot x + (-4)^2 = x^2 - 8x + 16$)

Rewrite eq: $\underbrace{(x-4)^2 - 16}_{} - 33 = 0$

$(x-4)^2 = 33 + 16 = 49$

so $x - 4 = 7$ or $x - 4 = -7$

$\underline{x = 11}$ $\underline{x = -3}$

b) $x^2 + 2x = (x+1)^2 - 1^2$ so

rewrite the eq: $(x+1)^2 - 1 = 63$

so $(x+1)^2 = 64$

$x+1 = 8$ or $x+1 = -8$

$\underline{x = 7}$ or $\underline{x = -9}$

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4. Equations with given solutions

If r_1 and r_2 are solutions ('roots') to the quadratic equation $x^2 + bx + c = 0$

then $(x - r_1)(x - r_2) = x^2 \underbrace{- r_2 x - r_1 x} + (-r_1)(-r_2)$

$$= x^2 - (r_1 + r_2)x + r_1 r_2$$

$$= x^2 + bx + c$$

so $b = -(r_1 + r_2)$

and $c = r_1 r_2$

Ex $x^2 + 6x - 16 = (x-2)(x+8)$ $\quad r_1 = +2$, $r_2 = -8$

Problem Determine the ~~expression~~ quad. expression $x^2 + bx + c$ with the given roots.

a) 1 and 2 $\quad (x-1)(x-2) = x^2 - 3x + 2$

b) 11 and −3 $\quad (x-11)(x+3) = x^2 - 8x - 33$

Ex $3(x-1)(x-2) = 3x^2 - 9x + 6 = 3(x^2 - 3x + 2)$