EBA 2911, lecture 7, 29 Sept. 2021, Runar Ile

Plan
1. Functions and graphs
2. Linear functions and straight lines
3. Quadratic functions and parabolas
4. Revenue- and cost functions

1. Functions and graphs

Ex
- Empirical functions
- the temperature as a function of time
- fertility
- the price of salmon
- all kinds of 'indexes'

A function is a table of function values

x	- - - - -
$f(x)$	- - - - -

Ex $f(x)$ = average age at first child birth in year x.

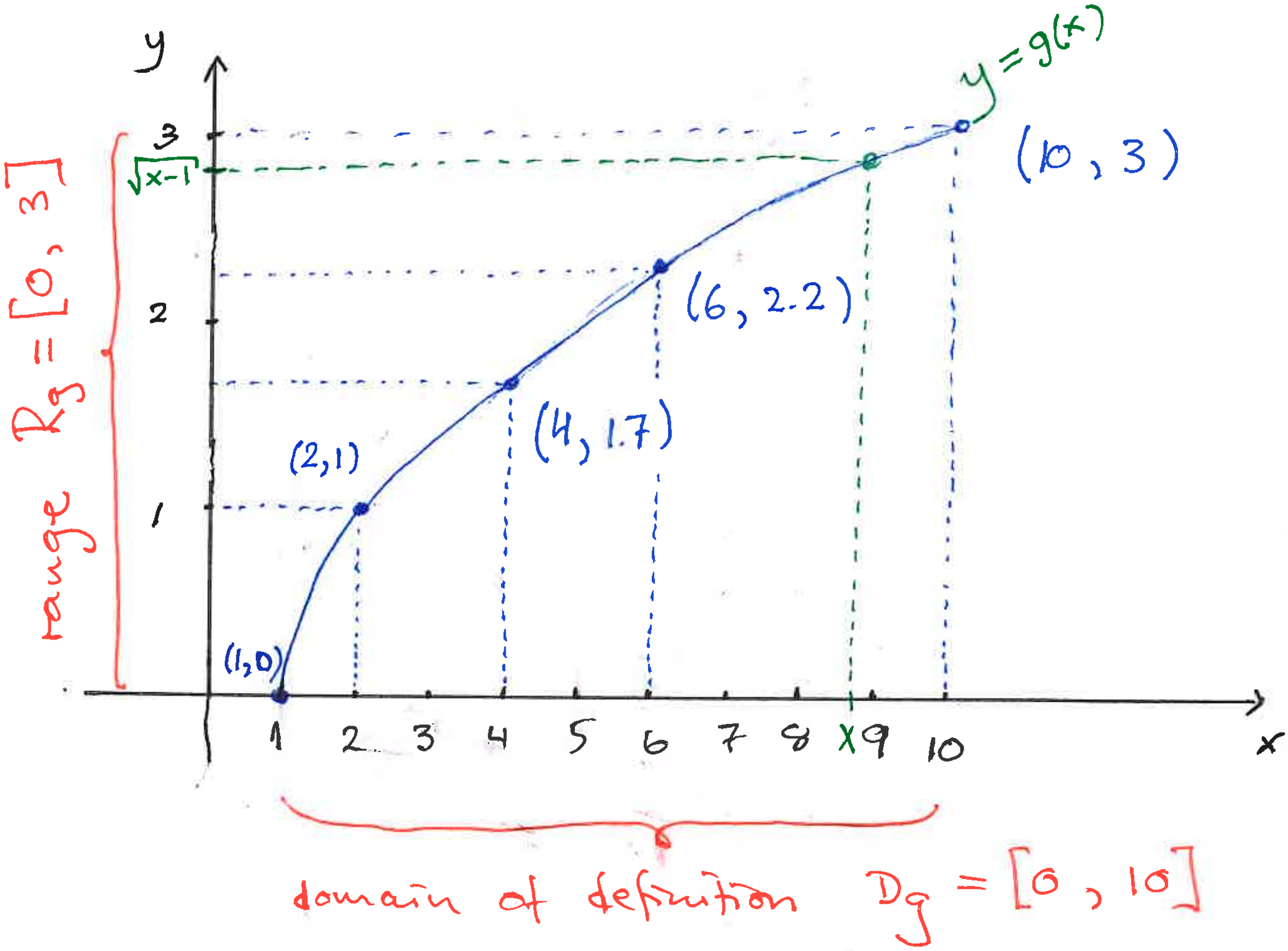
Domain of definition: $x \in [1961, 2019] = D_f$

Ex $g(x) = \sqrt{x-1}$. The largest possible domain of definition is $D_g = [1, \rightarrow\rangle$

Want to draw the graph with $D_g = [1, 10]$.

I make a table of function values

x	1	2	4	6	10
$g(x)$	0	1	1.7	2.2	3



2. Linear functions $f(x) = ax + b$

- the graph is a line

The point-slope formula.

If (x_0, y_0) is a point on the graph (a line!) and a is the slope, then

$$y - y_0 = a \cdot (x - x_0)$$

Ex If $(x_0, y_0) = (9, 25)$

and $(x_1, y_1) = (11, 31)$ are

two points on the line, then the

slope is

$$a = \frac{\text{change in y-value}}{\text{change in x-value}} = \frac{31-25}{11-9}$$

$$= \frac{6}{2} = 3$$

The point-slope formula gives

$$y - 25 = 3\cdot(x-9)$$

so $$y = 3x - 27 + 25$$

$$\underline{\underline{y = 3x - 2}}$$

Problem The graph of a linear function $f(x)$ passes through the points $(20, 46)$ and $(170, 16)$

a) Calculate the slope of the line

b) Determine the expression for $f(x)$

c) Determine the intersection points of the line with the x-axis and the y-axis.

Start: 11.00

Solution a) the slope : $\frac{16-46}{170-20} = \frac{-30}{150} = \underline{\underline{-0.20}}$

b) the point-slope formula with $(20, 46)$

gives $y - 46 = -0.20 \cdot (x - 20)$

that is $y = -0.20x + 4 + 46$

$= \underline{\underline{-0.20x + 50}} = f(x)$

c) intersection with y-axis: $(0, f(0)) = \underline{\underline{(0, 50)}}$

intersection with x-axis: $y = 0$ gives

the equation $-0.20x + 50 = 0$

so $-0.2x = -50 \quad | : -0.2$

$x = \frac{-50}{-0.2} = \underline{\underline{250}}$

so the point is $\underline{\underline{(250, 0)}}$

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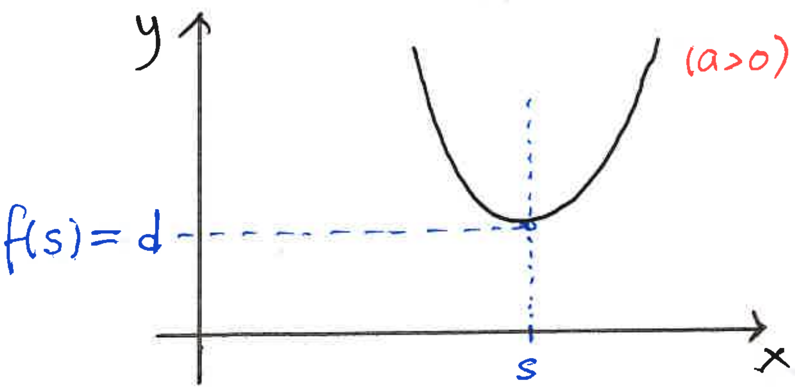
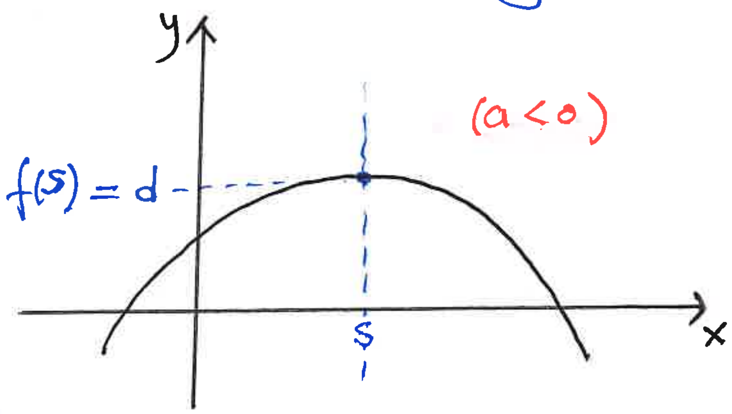
3. Quadratic functions

$f(x) = ax^2 + bx + c$

But if we want to draw/understand the graph, the following expression is better:

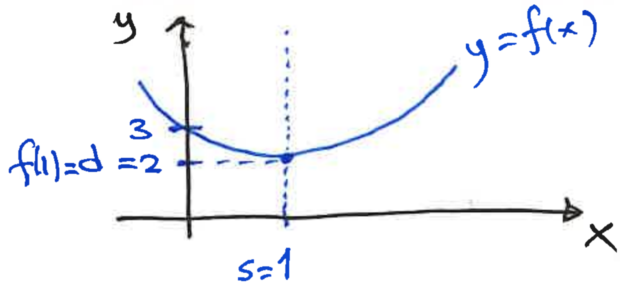
$f(x) = a(x-s)^2 + d$ "by completing the square"





EX $f(x) = x^2 - 2x + 3$

$= (x-1)^2 + 2$



Problem The quadratic function $f(x)$ has minimum value $y = -1$ and symmetry line $x = 5$ and the graph passes through the point $(9, 3)$.

a) Determine the expression $f(x) = a(x-s)^2 + d$

b) Determine where the graph of $f(x)$ crosses the x-axis and the y-axis.

Solution a) We have been given $s = 5$ and $d = -1$

so $f(x) = a(x-5)^2 - 1$

Then $f(9) = 3$ gives $a(9-5)^2 - 1 = 3$

$a \cdot 16 - 1 = 3$

so $16a = 4$

so $a = \frac{4}{16} = \frac{1}{4} = 0.25$

and $f(x) = \underline{\underline{0.25 \cdot (x-5)^2 - 1}}$

b) Crosses the x-axis: solve $f(x) = 0$

i.e. $0.25(x-5)^2 - 1 = 0$ $\quad 1 \cdot 4$

gives $(x-5)^2 = 4$

so $x - 5 = \pm 2$ so $\underline{\underline{x = 3}}$ or $\underline{\underline{x = 7}}$

Crosses the y-axis: $y = f(0) = 0.25(0-5)^2 - 1$

$= 0.25 \cdot 25 - 1 = 6.25 - 1 = \underline{\underline{5.25}}$

or as point $\underline{\underline{(0, 5.25)}}$

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4. Revenue- and cost functions

Profit = Revenue - Cost

$P(x) = R(x) - C(x)$, x = number of units produced and sold

Ex $R(x) = 15x$, $C(x) = 0.05x^2 - 10x + 525$

Determine the number of units x which maximizes profit and calculate max. profit.

$$P(x) = -0.05x^2 + 25x - 525$$

$$= -0.05 \cdot \left[x^2 + \frac{25x}{-0.05} + \frac{-525}{-0.05} \right]$$

$$= -0.05 \cdot \left[x^2 - 500x + 10500 \right]$$

$$= -0.05 \cdot \left[(x-250)^2 - 250^2 + 10\,500 \right]$$

$$= -0.05 \cdot \left[(x-250)^2 - 52\,000 \right]$$

So max. profit if $\underline{\underline{x = 250}}$

Max. profit $= P(250) = -0.05 \cdot (-52\,000)$

$$= \frac{52000}{20} = \underline{\underline{2600}}$$