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1 Functions and graphs

Defn: A function f is a rule that assigns a value $f(x)$ to any point x in the domain of definition D_f.
The graph of f is the set of all pts (x, y) such that x is in D_f and $y = f(x)$.

Examples:

* Empirical functions: salmon prices

Table of function values:

x	1	2	3	4	....
$f(x)$	63	....			

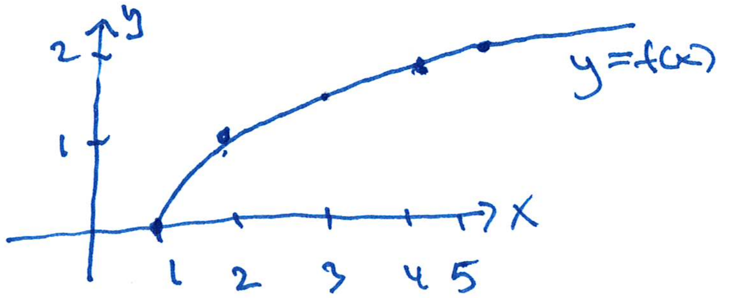
all x-values in the domain of defn.

all function values

* Mathematical function: $f(x) = \sqrt{x-1}$, $x \geq 1$

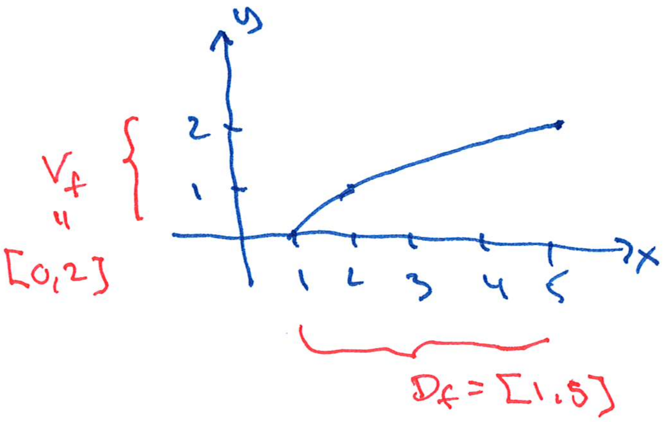
largest possible domain of defn. $D_f = [1, \infty)$

x	1	2	3	4	5	...
$f(x)$	0	1	$\simeq 1.4$	$\simeq 1.7$	2	--



Defn: The range V_f is the set of all function values $f(x)$ that you can obtain for points x in D_f.

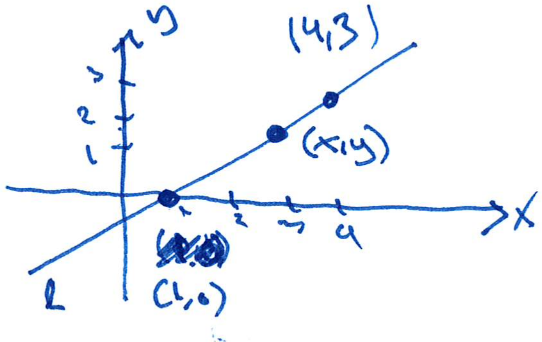
Ex: $f(x)=\sqrt{x-1}$, $D_f = [1,5]$



(2) Linear functions and straight lines

Defn: A linear function has the form $f(x) = ax+b$

The graph of f is a straight line $\Leftrightarrow$ f is linear



slope: $\dfrac{\Delta y}{\Delta x} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{3-0}{4-1} = \underline{\underline{1}}$

when l is a straight line, the slope is independent of the points on l you choose

$$1 = \frac{y-0}{x-1} = \frac{y}{x-1} \Leftrightarrow 1\cdot(x-1) = y \Leftrightarrow y = \underline{x-1}$$

$$a = \frac{y-y_0}{x-x_0} \Leftrightarrow a(x-x_0) = y-y_0 \Leftrightarrow \boxed{y-y_0 = a(x-x_0)}$$

one-point formula for a straight line with slope a that goes through (x_0,y_0)

$$y = y_0 + a(x-x_0)$$

graph of: $f(x) = y_0 + a(x-x_0)$

$= ax + (y_0 - ax_0)$

Summary:

the graph of $f(x) = ax + b$ is a straight line with slope a and y-intercept b.

Problem: Find the equation for the line through $(1,4)$ and $(3,-1)$.

Solution: Slope: $a = \frac{-1-4}{3-1} = \frac{-5}{2} = -2.5$

y-intercept: $b = \cancel{4}\ 6.5$

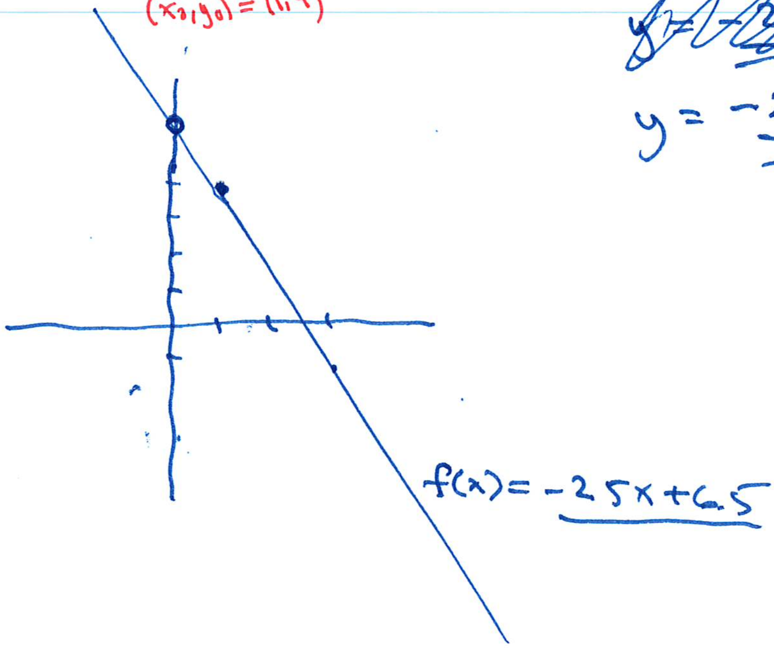
one-point formula with $a = -2.5$, $(x_0, y_0) = (1,4)$

$$y - 4 = -2.5(x-1)$$

$$y - 4 = -2.5x + 2.5$$

$$\cancel{y = -2.5x + 14}$$

$$y = -2.5x + 6.5$$



Note: vertical lines $x = x_0$ are **not** the graph of a function.

(3) Quadratic functions and parabolas

Defn: A quadratic function has the form
$f(x) = ax^2+bx+c$ with $a \neq 0$.
The graph of a quadratic function is called a parabola.

Ex: $f(x) = x^2-4x+3$
$= \underbrace{x^2-4x+4}_{\text{square}} + 3 - 4$

completing the square
$x^2+px+\left(\frac{p}{2}\right)^2$
$= \left(x+\frac{p}{2}\right)^2$

$f(x) = \underline{(x-2)^2 - 1}$

Ex: $f(x) = 4x^2-12x+8$
$= 4(x^2-3x+2) = 4(\underbrace{x^2-3x+(3/2)^2}_{(x-3/2)^2}+2-(3/2)^2)$

$= 4(x-3/2)^2 + \underbrace{4(2-9/4)}_{8-9}$
$= \underline{4(x-3/2)^2 - 1}$ ← $a=4$, $s=3/2$, $d=-1$

Alt: $f(x) = 4x^2-12x+8$
$= \underline{(2x-3)^2-1} = [2\cdot(x-3/2)]^2-1$
$= \underline{4\cdot(x-3/2)^2-1}$

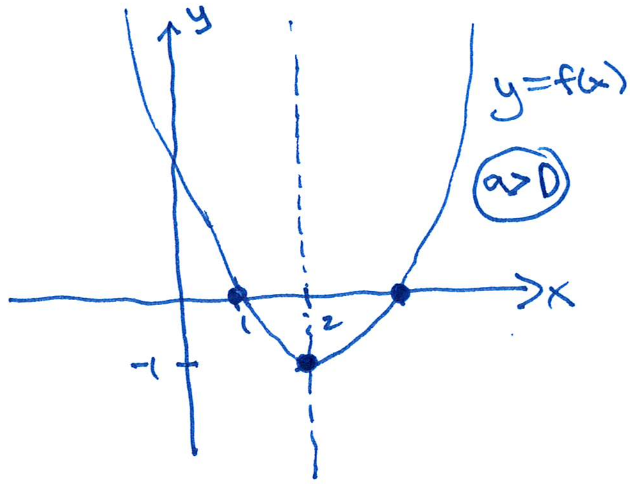
Fact: $\boxed{\text{Any quadratic function } f(x)=ax^2+bx+c \ (a\neq 0) \text{ can be written } f(x) = a\cdot(x-s)^2+d.}$

Ex: $f(x) = \underline{x^2 - 4x + 3} = \underline{(x-2)^2 - 1}$

$\underline{a=1}$ $\underline{b=-4}$ $\underline{c=3}$ $\qquad a=1$ $s=2$ $d=-1$

Symmetry line: $\underline{x = s}$

point on the symmetry line: $\begin{cases} x = s \\ y = d \end{cases}$



$x = s$

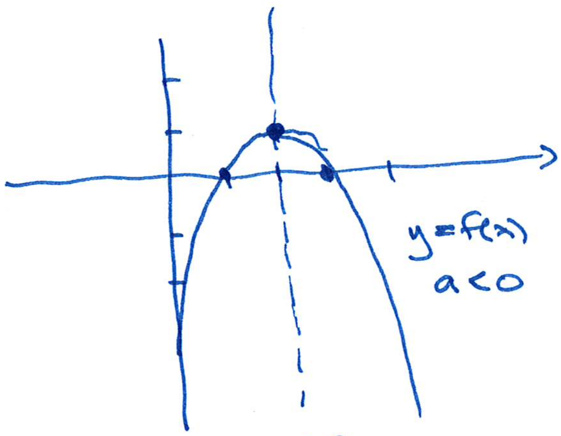
$x = 2$

Ex: $f(x) = -x^2 + 4x - 3$

$= -(x^2 - 4x + 3) = -((x-2)^2 - 1)$

$= \underline{-(x-2)^2 + 1}$

$a = -1 \quad s = 2 \quad d = 1$



$x = 2$

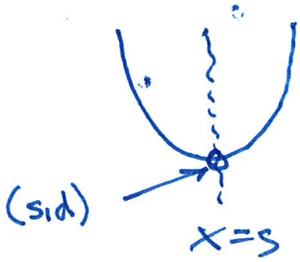
Summary: $f(x) = ax^2 + bx + c \quad (a \neq 0)$ quadr. function

$$= a(x-s)^2 + d$$

$a > 0$:

symmetry line $x = s$

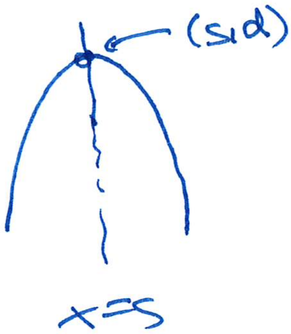
minimum pt (s, d)



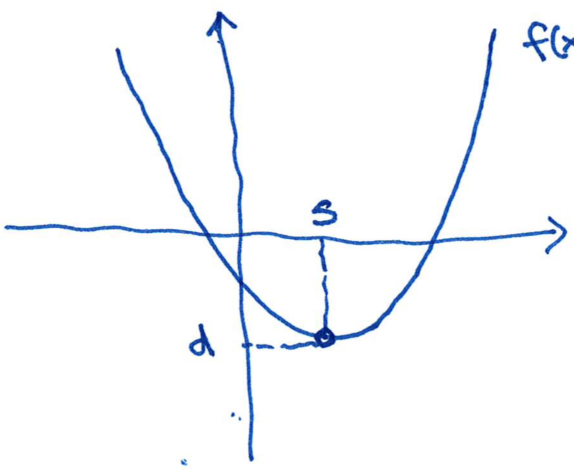
$a < 0$:

symmetry line $x = s$

maximum pt (s, d)



Ex:



$f(x)$ quadr. function

$= a(x-s)^2 + d$

4 Revenue and cost functions

$$P(x) = \underset{\text{revenue}}{R(x)} - \underset{\text{cost}}{C(x)}$$

x units produced and sold

Ex: $R(x) = 15x$ price per unit $= 15$

$$C(x) = 0.05x^2 - 10x + 525$$

$$= (0.05x - 10)x + 525$$

$$P(x) = 15x - (0.05x^2 - 10x + 525), \quad x \geq 0$$

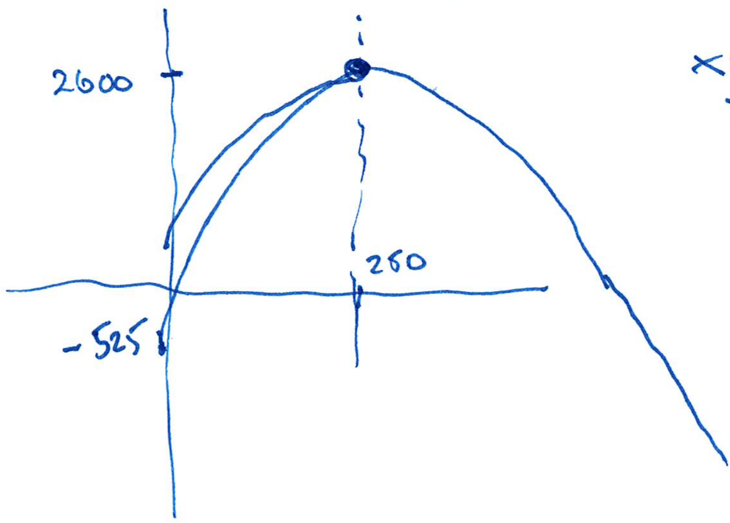
$$= -0.05x^2 + 25x - 525$$

$$= -0.05(x^2 - 500x \quad \cancel{} + 10500)$$

$$= -0.05(x^2 - 500x + 250^2) - 0.05(10500 - 250^2)$$

$$= -0.05(x - 250)^2 + \frac{250^2}{20} - 525$$

$$= \underline{-0.05(x-250)^2 + 2600}$$



$\underline{\underline{x = 250}}$ gives the maximal profit $\underline{\underline{2600}}$