EBA 1180, lecture 12, 5 Oct. 2022, Runar Ile

<u>Plan</u>

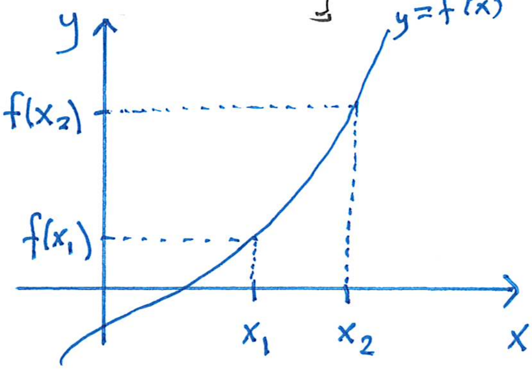
1. Increasing and decreasing functions
2. Circles and ellipses
3. Polynomial functions

1. Increasing and decreasing functions

<u>Definition</u> A function $f(x)$ is increasing

if for all $x_1 < x_2$

one has $f(x_1) \leq f(x_2)$



<u>Ex</u> $f(x) = 2x+5$ is increasing for all x.

Reason: Assume $x_1 < x_2 \quad | \cdot 2$

$$2x_1 < 2x_2 \quad | + 5$$

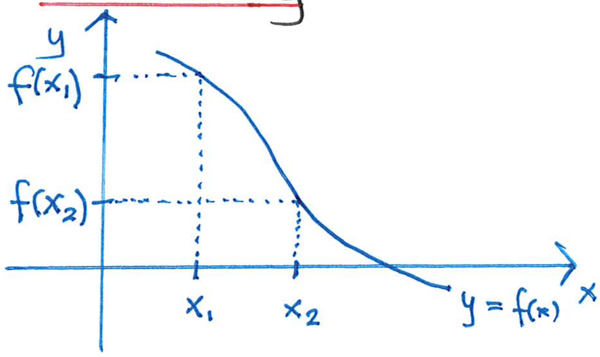
$$f(x_1) = 2x_1+5 < 2x_2+5 = f(x_2)$$

so $f(x)$ is increasing (actually strictly increasing)

<u>Definition</u> A function $f(x)$ is decreasing

if for all $x_1 < x_2$

one has $f(x_1) \geq f(x_2)$



<u>Problem</u> Show that $f(x) = -2x+5$ is (strictly) decreasing.

Solution Suppose $x_1 < x_2 \quad | \cdot (-2)$

$$-2x_1 > -2x_2 \quad | +5$$

$$f(x_1) = -2x_1 + 5 > -2x_2 + 5 = f(x_2)$$

Problem We have a constant function $f(x) = 5$

Decide whether $f(x)$ is increasing/decreasing/neither.

Solution

Increasing: If $x_1 < x_2$ then $f(x_1) = 5 \leq 5 = f(x_2)$

Decreasing: If $x_1 < x_2$ then $f(x_1) = 5 \geq 5 = f(x_2)$

– so both.

But: Not strictly increasing or strictly decreasing

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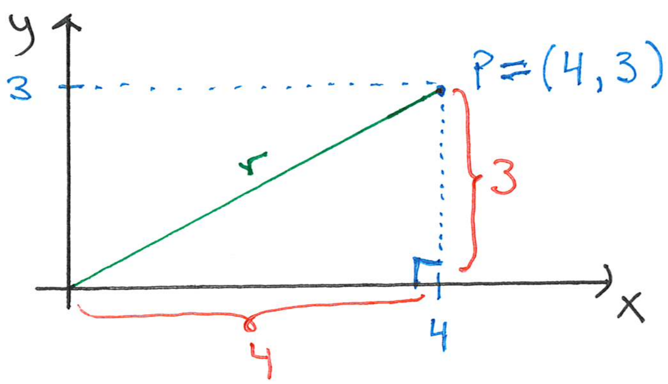
2. Circles and ellipses

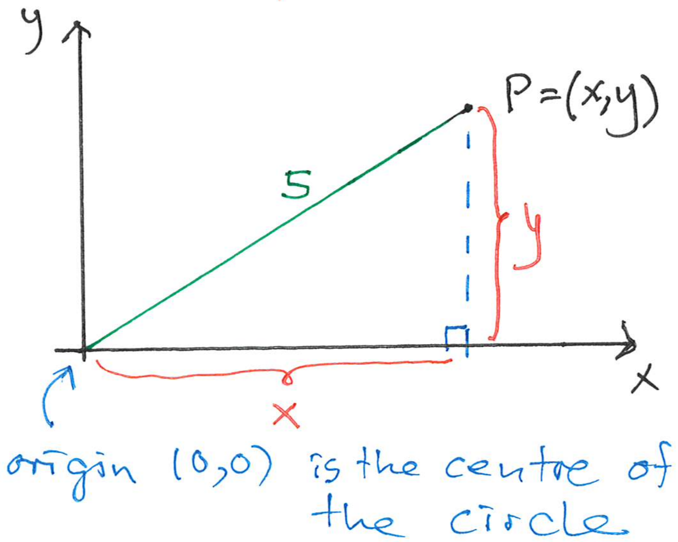


Pytagoras:

$$r^2 = 4^2 + 3^2 \quad (r \geq 0)$$

$$r^2 = 16 + 9 = 25$$

$$r = \sqrt{25} = 5$$



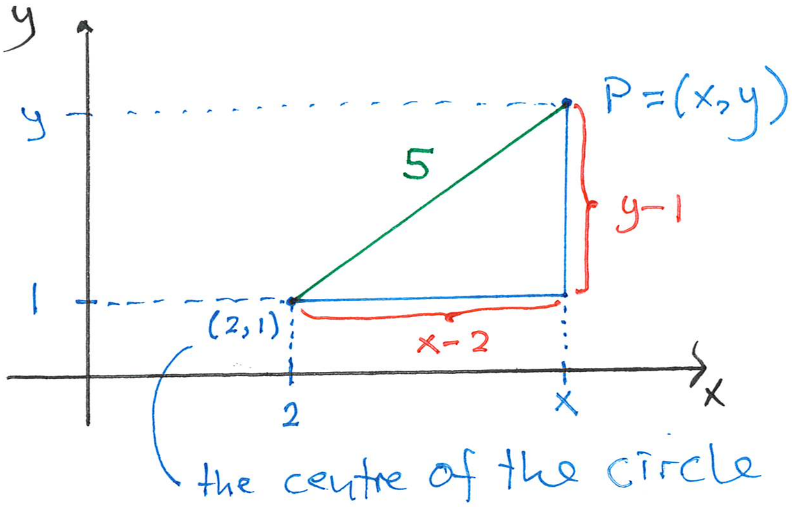
Pytagoras:

$$25 = x^2 + y^2$$

- one equation
- two unknowns
- infinitly many solutions

The solutions are the points on a circle with radius 5 and centre $(0, 0)$.

Start : 11.02



Pytagoras:

$$5^2 = (x-2)^2 + (y-1)^2$$

$$25 = x^2 - 4x + 4 + y^2 - 2y + 1$$

that is:

$$x^2 + y^2 - 4x - 2y = 20$$

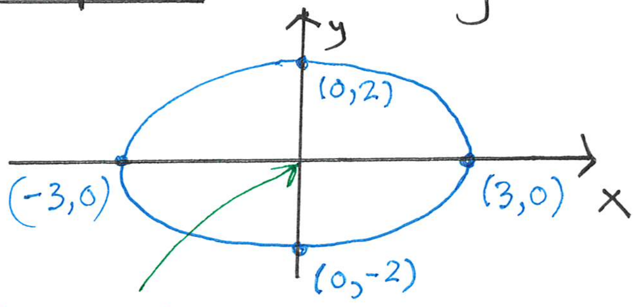
Problem Determine the radius and the centre of $x^2 + y^2 - 2x + 6y = -9$

Solution $\underbrace{(x-1)^2}_{x^2-2x+1} + \underbrace{(y+3)^2}_{y^2+6y+9} = -9 + 1 + 9 = 1$

Centre : $(1, -3)$, radius : $\sqrt{1} = 1$

Ellipses $4x^2 + 9y^2 = 36$

x	3	−3	0	0
y	0	0	2	−2



the centre of the ellipse : $(0,0)$

I divide each side by 36 :

$$\underbrace{\frac{4}{36}}_{=\frac{1}{9}} x^2 + \underbrace{\frac{9}{36}}_{=\frac{1}{4}} y^2 = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

- similar to a circle equation

but the x-axis is stretched by a factor 3

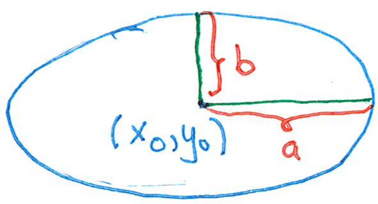
and the y-axis ——— " ——— 2

In general, any ellipse is the set of solutions of an equation of the form

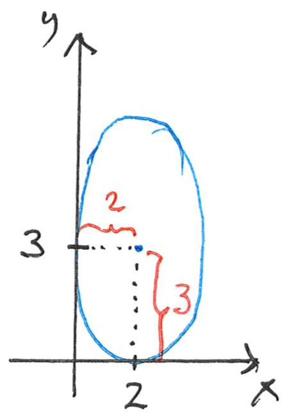
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

Here (x_0, y_0) is the centre and

a and b are the semi-axes



Ex $$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$$



Centre: $(2, 3)$

Semi-axes: $a = \sqrt{4} = 2$ and $b = \sqrt{9} = 3$.

3. Polynomial functions

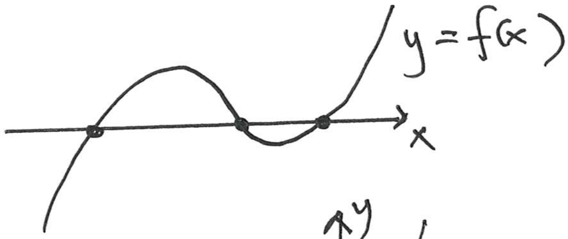
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \qquad (a_n \neq 0)$$

has degree $\deg(f) = n$

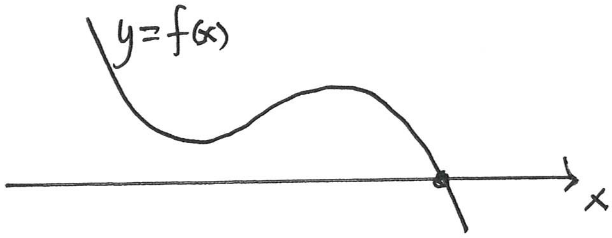
Then :

- $f(x)$ has at most n roots (zeros)
- If the degree is an odd number then $f(x)$ has at least one root.

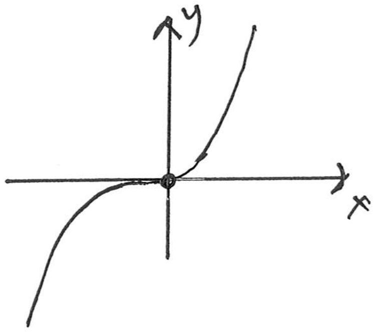
Ex



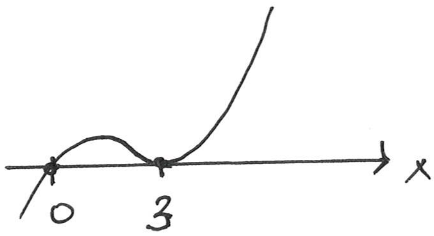
has degree 3



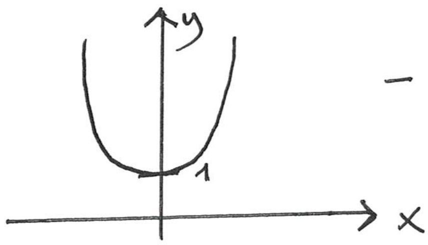
$y = x^3$:



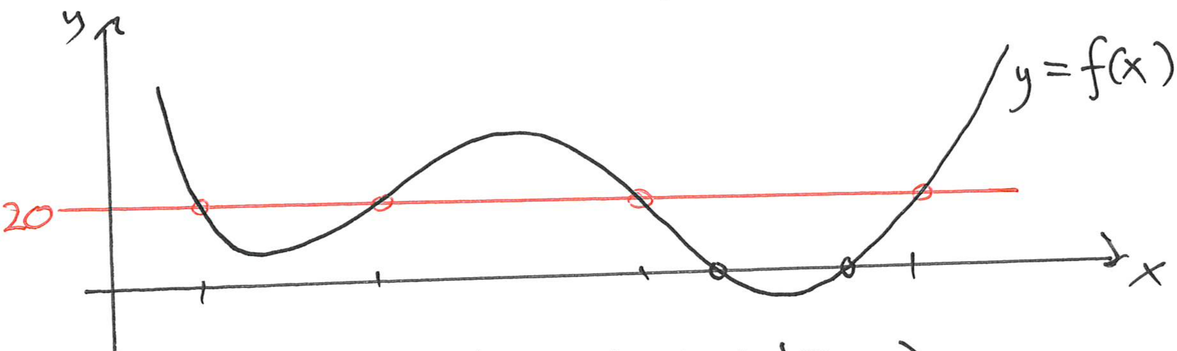
$y = x(x-3)^2$:



Ex $f(x) = x^4 + 1$



– no roots !



degree 4

$f(x) = 20$ has 4 roots (solutions)

$f(x) - 20 = 0$ ——— " ———

still a polynomial of the same degree as $f(x)$.