EBA1180, lecture 14, 12 oct. 2022, Runar Ile

Plan
1. Inverse functions
2. Exponential functions
3. Logarithms

1. Inverse functions

What is a functions?
- an expression
- a table of function values
- a graph
- a situation

Ex $f(x) = (x-3)^2$

with domain $D_f = [3, \rightarrow\rangle$

(so $x \geq 3$)

Table of function values

x	3	4	5	6	7	....	$g(x)$
$f(x)$	0	1	4	9	16	....	x

← inverse function of $f(x)$.

so $g(0) = 3$, $g(1) = 4$, $g(4) = 5$, ...

$f(g(0)) = f(3) = 0$ and $g(f(3)) = g(0) = 3$

$f(g(1)) = f(4) = 1$ and $g(f(4)) = g(1) = 4$

$f(g(4)) = f(5) = 4$ and $g(f(5)) = g(4) = 5$

Definition $f(x)$ with domain D_f and $g(x)$ with domain D_g are inverse functions if

$f(g(x)) = x$ for all x in D_g and $g(f(x)) = x$ for all x in D_f

If so, the domain of $g(x)$ is the range of $f(x)$. Short: $D_g = R_f$

Also: $f(x)$ is the inverse function of $g(x)$

so $R_g = D_f$

How to find an expression for the inverse function?

① Solve the equation $y = f(x)$ for x

② Switch the variables x and y

③ Put $D_g = R_f$ and determine R_f.

Ex $f(x) = (x-3)^2$ with $D_f = [3, \rightarrow\rangle$

we find D_g by ① - ③.

① Solve the equation

$$y = (x-3)^2 \quad \text{for all } x \text{ in } D_f$$

- take the square root on each side

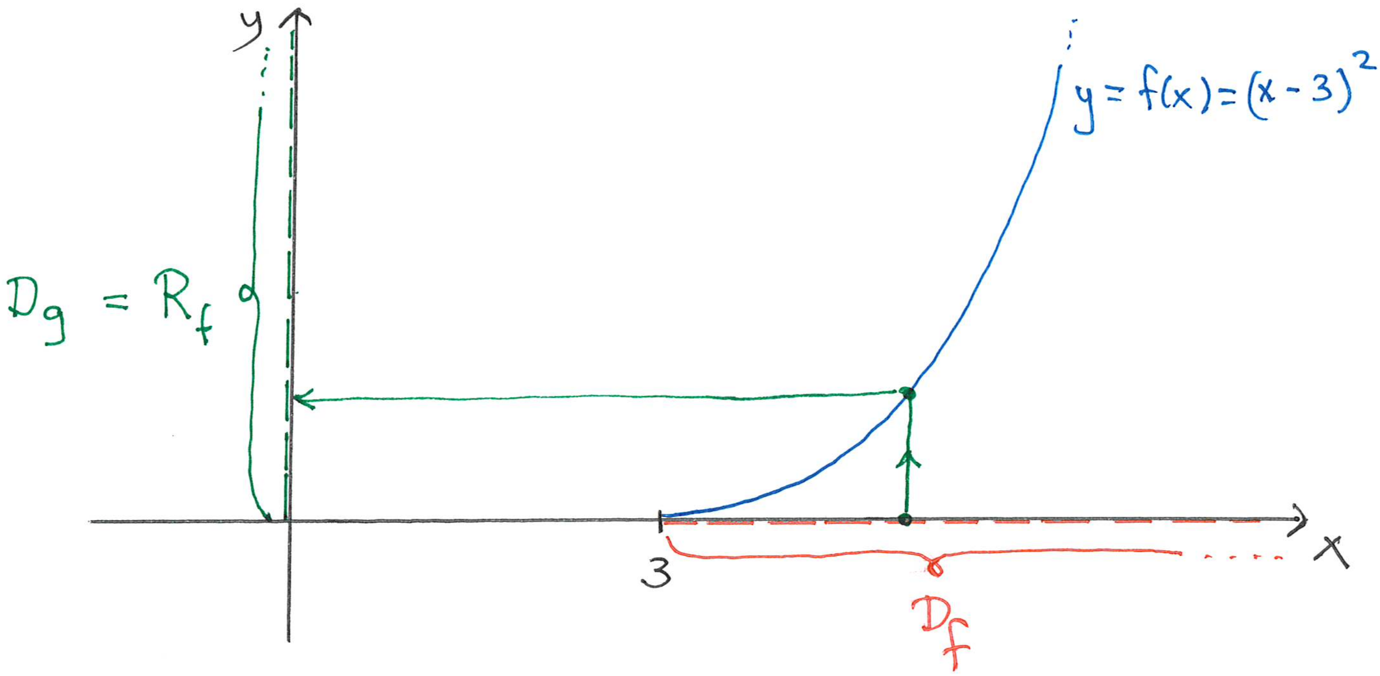
$$\sqrt{y} = |x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

So $\sqrt{y} = x - 3$ since $x \in D_f = [3, \rightarrow\rangle$

and then $x = \underline{3 + \sqrt{y}}$

② Switch variables: $y = g(x) = 3 + \sqrt{x}$

③ $D_g \overset{\text{always}}{=} R_f = [0, \rightarrow\rangle$ because $f(x) = (x-3)^2 = y$ has a solution x with $x \geq 3$ for all values $y \geq 0$



Note that $f(g(x)) = ((3+\sqrt{x}) - 3)^2 = x$

and $g(f(x)) = 3 + \sqrt{(x-3)^2} = x$ (since $x \geq 3$)

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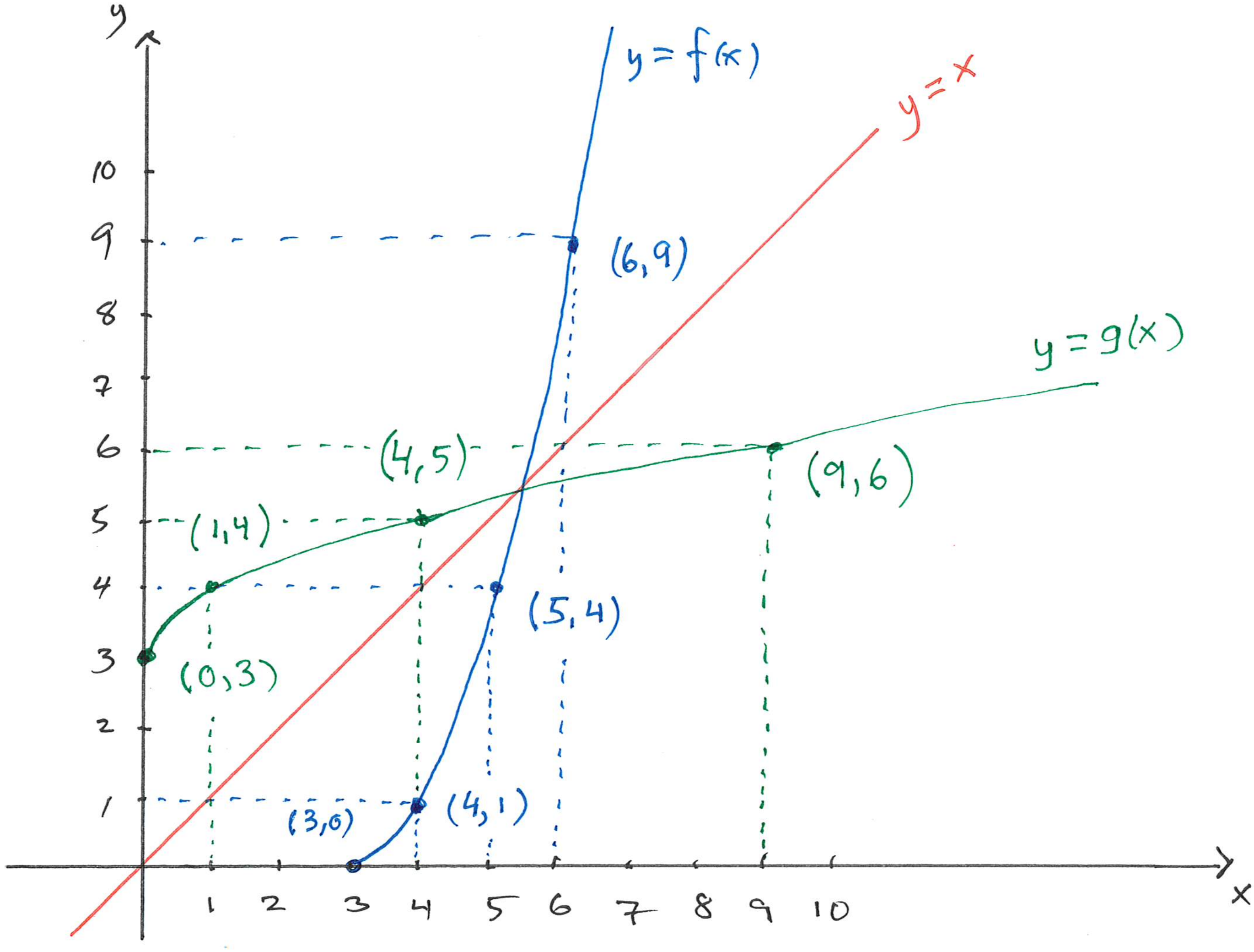
<u>The graph of the inverse function</u>

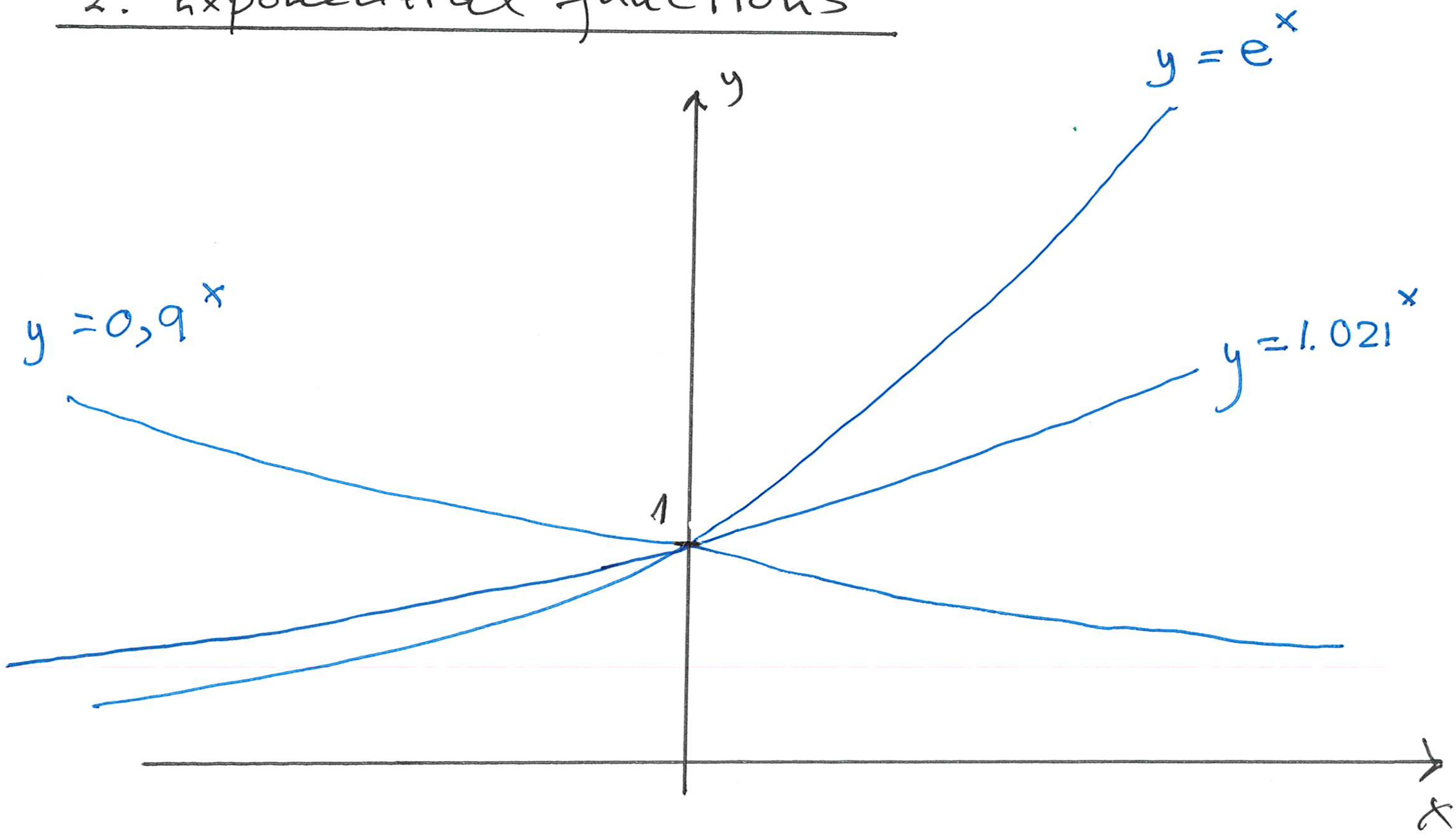
- is the mirror image of the graph of $f(x)$ with respect to the "diagonal" $y = x$

<u>Ex</u> $f(x) = (x-3)^2$ with $D_f = [3, \rightarrow\rangle$

x	3	4	5	6	7	...	$g(x)$
$f(x)$	0	1	4	9	16	...	x



2. Exponential functions



$a > 1$ $\quad f(x) = a^x$ is strictly increasing without upper bounds

and $\quad a^x \xrightarrow[x \to -\infty]{} 0^+$

$0 < a < 1$ $\quad f(x) = a^x$ is strictly decreasing without upper bounds

and $\quad a^x \xrightarrow[x \to \infty]{} 0^+$

Both cases $\quad D_f$ = all numbers on the number line $(= \mathbb{R})$

and $R_f = \langle 0, \rightarrow \rangle$

Power rules $\quad$ If $f(x) = a^x$ then

$$f(x) \cdot f(y) = a^x \cdot a^y = a^{x+y} = f(x+y)$$

and $$\frac{1}{f(x)} = \frac{1}{a^x} = a^{-x} = f(-x)$$

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3. Logarithms $\quad$ Suppose $a > 0$ (and $a \neq 1$)

Then $g(x) = \log_a(x)$ is the inverse function of $f(x) = a^x$ and

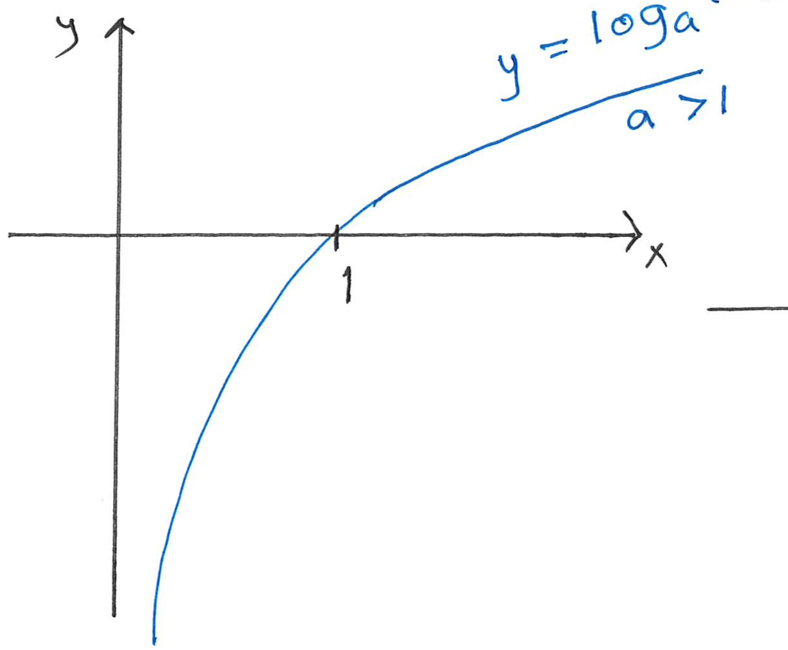
$$D_g = R_f = \langle 0, \rightarrow \rangle$$

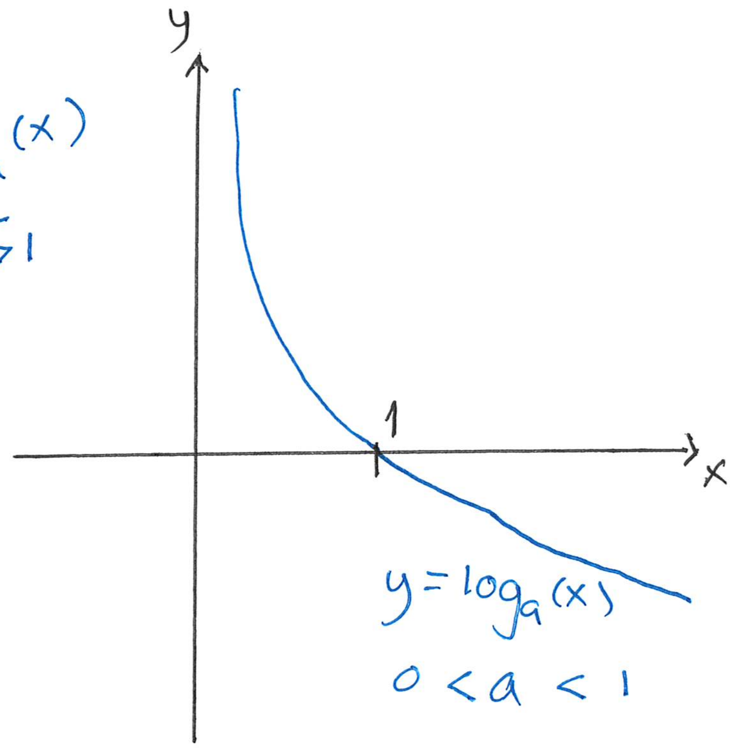
Ex $a = 2$, $\log_2(10)$ = the number which 2 has to be raised to to give 10

We have $2^{3.322} \simeq 10$

so $\log_2(10) \simeq 3.322$

Graphs





the y-axis $(x = 0)$ is a vertical asymptote in both cases.

Rules
$$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$
$$\log_a(x^r) = r \cdot \log_a(x).$$

Definition $\ln(x) = \log_e(x)$, e = Euler number

– is called the natural logarithm

$\ln(x)$ is the inverse function of e^x

so $e^{\ln(x)} = x$ and $\ln(e^x) = x$