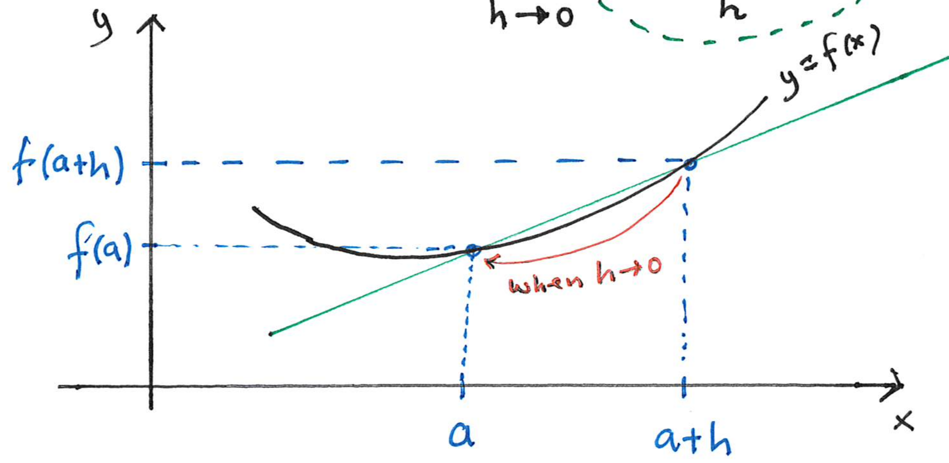
EBA 1180, lecture 17, 1 Nov. 2022, Runar Ile

Plan: Repetition of differentiation

1. Definition, slopes and graphs
2. The natural logarithm
3. Rules of differentiation

1. Rep. of definition, slopes and graphs

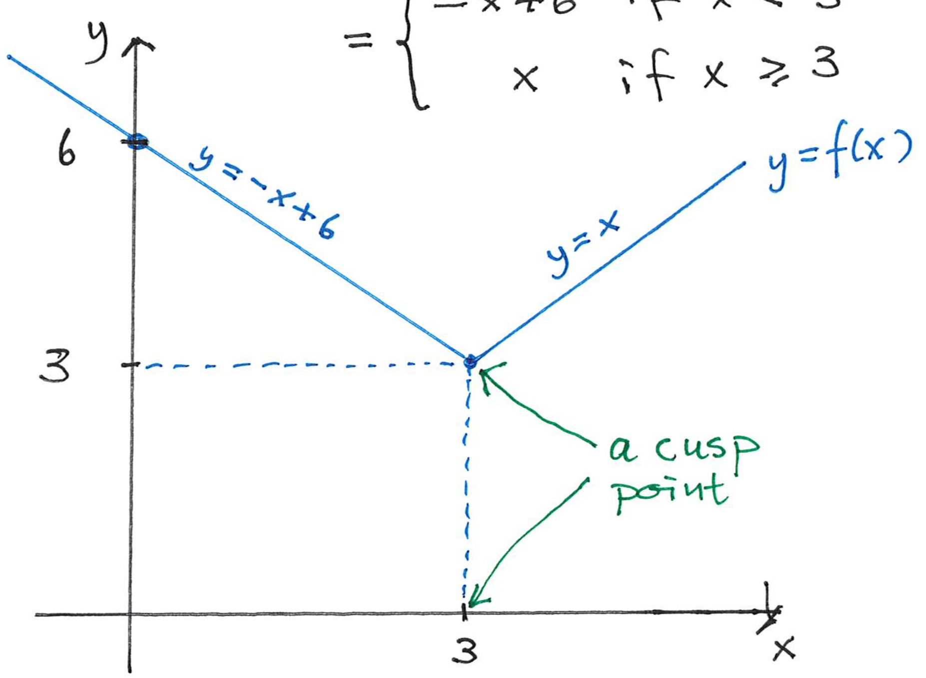
Definition $\quad f'(a) = \lim\limits_{h\to 0} \frac{f(a+h)-f(a)}{h}$ = the slope of the secant



Note The derivative does not always exist!

EX $\quad f(x) = |x-3|+3 = \begin{cases} -(x-3)+3 & \text{if } x<3 \\ x-3+3 & \text{if } x \geq 3 \end{cases}$

$$= \begin{cases} -x+6 & \text{if } x<3 \\ x & \text{if } x \geq 3 \end{cases}$$



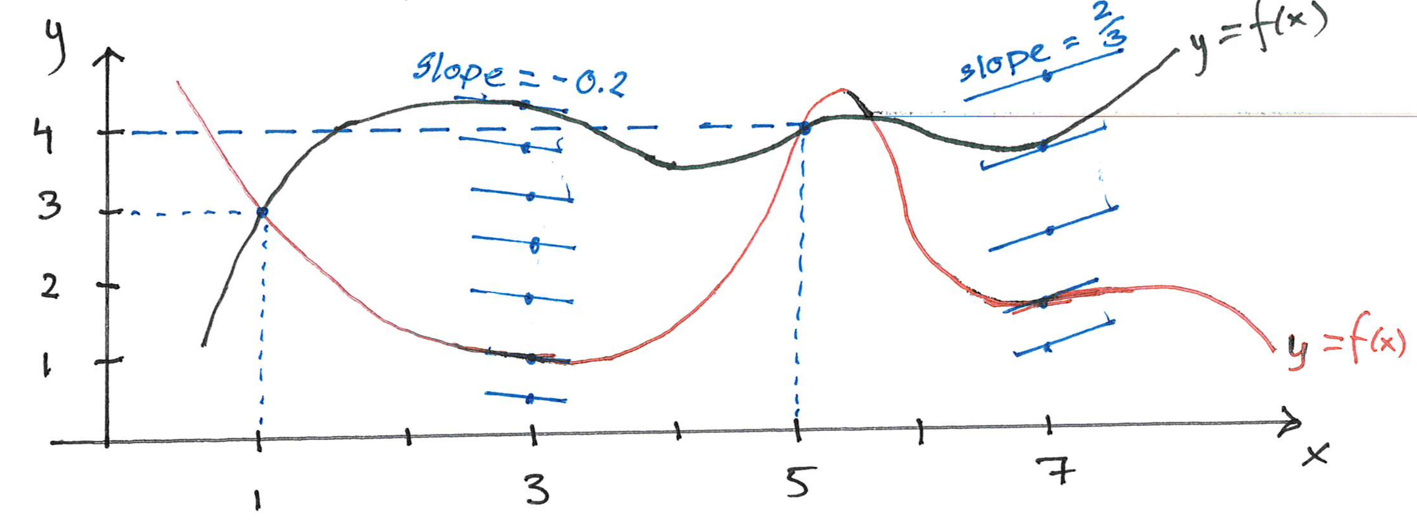
Here

$$f'(x) = \begin{cases} -1 & \text{if } x<3 \\ 1 & \text{if } x>3 \end{cases}$$

But for $x=3$ there is no tangent: hence $f'(3)$ does not exist.

Probl. 1d from last week. Sketch two graphs.

$$f(1) = 3, \quad f'(3) = -0.2, \quad f(5) = 4, \quad f'(7) = \frac{2}{3}$$



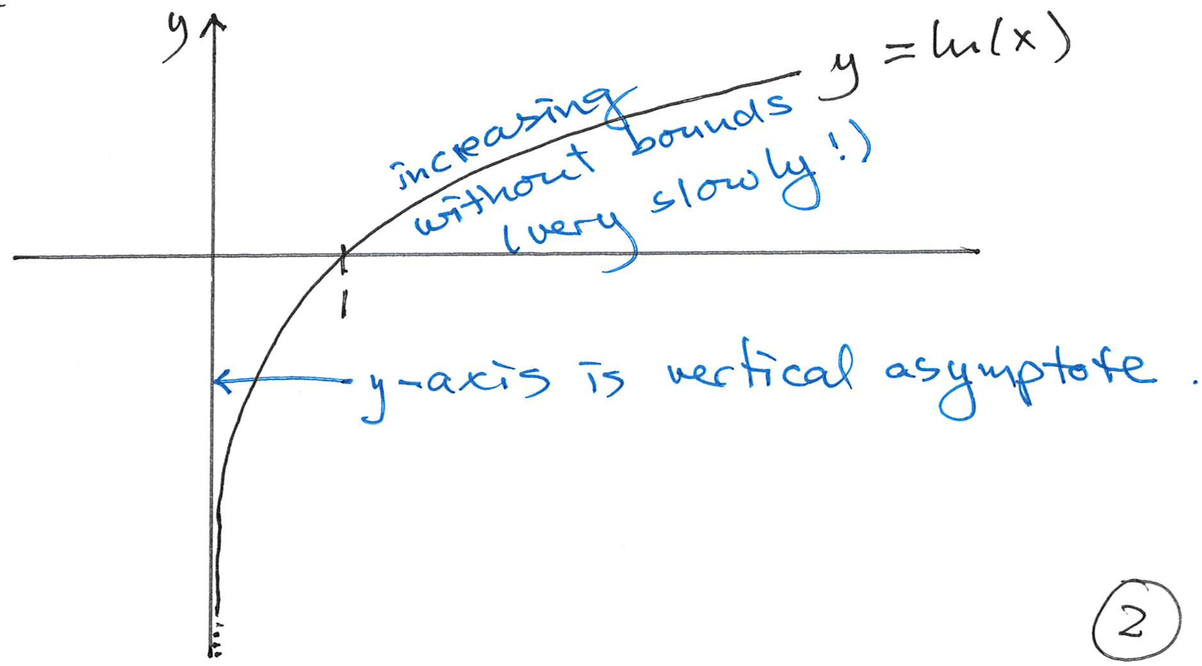
2. The natural logarithm

$\ln(x)$ is the inverse function of e^x

so $\ln(e^x) = x$ and $e^{\ln(x)} = x$

Domain of definition for $\ln(x)$ is range of e^x, that is: all positive numbers

The range of $\ln(x)$ is the domain of e^x: the whole number line



Ex $\ln(\sqrt[10]{e}) = \ln(e^{\frac{1}{10}}) = \frac{1}{10}\ln(e) = \frac{1}{10}\cdot 1$

$\overset{\ln(e^x)=x}{=} \frac{1}{10}$

$\ln(3e) = \ln(3) + \ln(e) = \ln(3)+1$

$e^{2\ln(5)} = e^{\ln(5^2)} = 5^2 = 25$

$= (e^{\ln(5)})^2 = 5^2$

$e^{\ln(2)+\ln(3)} = e^{\ln(2)} \cdot e^{\ln(3)} = 2\cdot 3 = 6$

Note: $\ln(2+3) \neq \ln(2) + \ln(3)$

$\ln(2+3) = 1.6094$, $\quad \ln(2)+\ln(3) = 0.6931 + 1.0986 = 1.7918$

Ex $\ln(5x) = \ln(5) + \ln(x)$

$\ln(x^{10}) = 10\ln(x)$

$\ln\left(\frac{3}{(x-1)}\right) = \ln(3) - \ln(x-1)$

Start: 11.00

3. Rules of differentiation

Product rule : $\left[g(x)\cdot h(x)\right]' = g'(x)\cdot h(x) + g(x)\cdot h'(x)$

Ex $\left[(x^2+1)\cdot e^x\right]' = (x^2+1)'\cdot e^x + (x^2+1)\cdot(e^x)'$

$= 2x\cdot e^x + (x^2+1)\cdot e^x$

$= \underline{\underline{(x^2+2x+1)\,e^x}}$ — zero ?

Probl $\left[\sqrt{x}\cdot \ln(x)\right]' = (x^{\frac{1}{2}})'\cdot \ln(x) + x^{\frac{1}{2}}\cdot\left[\ln(x)\right]'$

$= \frac{1}{2}x^{\frac{1}{2}-1}\cdot \ln(x) + x^{\frac{1}{2}}\cdot x^{-1}$ — zero ?

$= \frac{1}{2}x^{-\frac{1}{2}}\cdot \ln(x) + x^{-\frac{1}{2}}$

$= x^{-\frac{1}{2}}\left(\frac{1}{2}\ln(x) + 1\right) \quad \Big|\cdot\frac{2}{2} = 1$

$= \underline{\underline{\dfrac{\ln(x)+2}{2\sqrt{x}}}}$

$\underline{\text{zero}}$: $\ln(x)+2=0$

$\ln(x) = -2$

$x = e^{\ln(x)} = \underline{\underline{e^{-2}}}$

$\underline{\text{pos}}$?

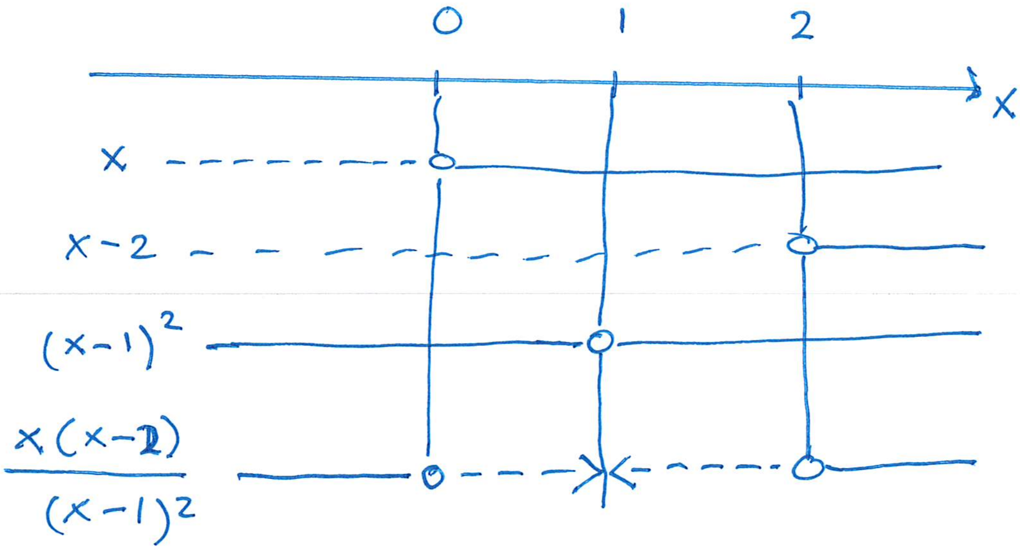
Quotient rule

$$\left[\frac{g(x)}{h(x)}\right]' = \frac{g'(x)\cdot h(x) - g(x)\cdot h'(x)}{\left[h(x)\right]^2}$$

Ex $\left[\dfrac{x^2}{x-1}\right]' = \dfrac{2x\cdot(x-1) - x^2\cdot 1}{(x-1)^2} = \dfrac{2x^2-2x-x^2}{(x-1)^2}$

$= \dfrac{x^2-2x}{(x-1)^2} = \underline{\underline{\dfrac{x(x-2)}{(x-1)^2}}}$ — pos ?

Sign diag:



Ex $\left[\frac{\ln(x)}{x}\right]' = \frac{\frac{1}{x}\cdot x - \ln(x)\cdot 1}{x^2}$

$g(x) = \ln(x)$

$g'(x) = \frac{1}{x}$

$h(x) = x$

$h'(x) = 1$

– zero ? $x = e$

– pos ? $0 < x < e$

$= \underline{\underline{\frac{1 - \ln(x)}{x^2}}}$

<u>Chain rule</u> $\left[g(u(x))\right] = g'(u)\cdot u'(x)$ where $u = u(x)$

Ex $\left[e^{x^2+3x}\right]' = e^u\cdot(2x+3) = \underline{\underline{(2x+3)\cdot e^{x^2+3x}}}$

$u(x) = x^2 + 3x$ and $g(u) = e^u$

$u'(x) = 2x + 3$ $\quad g'(u) = e^u$

Ex $\left[\ln(x^2+5)\right]' = \frac{1}{u}\cdot 2x = \underline{\underline{\frac{2x}{x^2+5}}}$

$u(x) = x^2 + 5$ and $g(u) = \ln(u)$

$u'(x) = 2x$ $\quad g'(u) = \frac{1}{u}$

Ex $\left[\ln\left(\frac{3x}{x-1}\right)\right]' = \left[\ln(3x) - \ln(x-1)\right]'$

$$= \left[\ln(3) + \ln(x) - \ln(x-1)\right]'$$

$$= 0 + \frac{1}{x} - \frac{1}{x-1} = \frac{x-1-x}{x(x-1)} = \frac{-1}{x(x-1)}$$

pos: $0 < x < 1$

$u(x) = x-1$ and $g(u) = \ln(u)$

$u'(x) = 1$ $\qquad g'(u) = \frac{1}{u}$