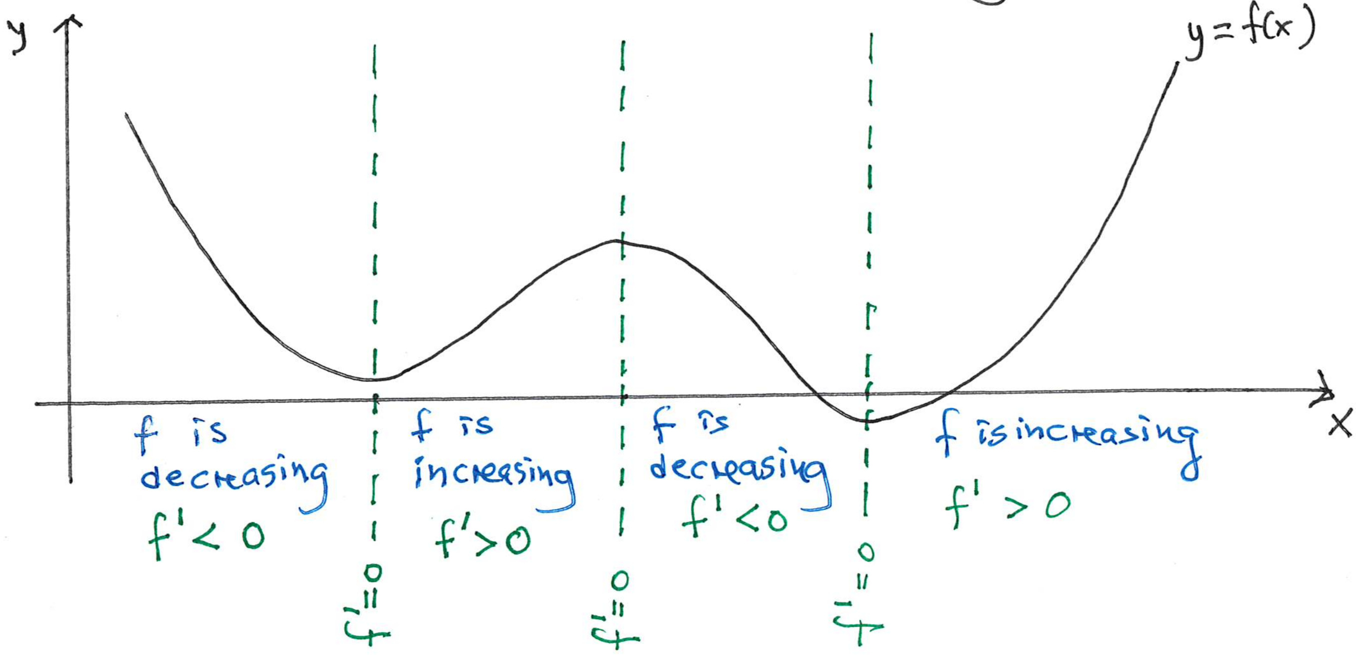
EBA1180, lecture 18, 2 Nov. 2022, Runar Ile

Plan

1. Local max/min and stationary points
2. Global max/min
3. The mean value theorem

1. Local max/min and stationary points



When $f'(x)$ is positive, $f(x)$ is increasing

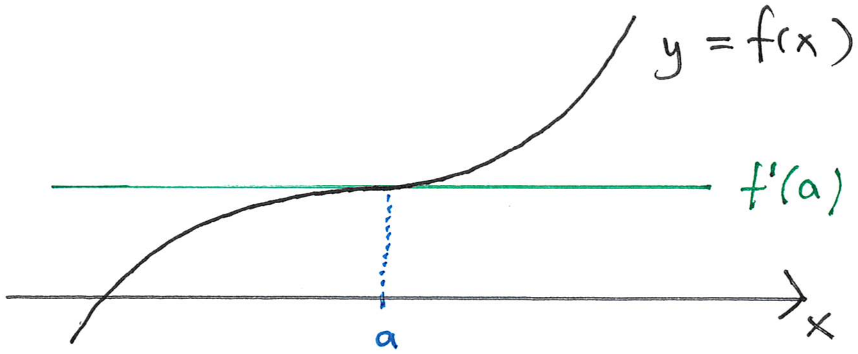
When $f'(x)$ is negative, $f(x)$ is decreasing

<u>Important conclusion</u> The sign diagram of $f'(x)$ determines where $f(x)$ is increasing and decreasing

If $x=a$ is a local minimum point, then $f'(a)=0$ <u>and</u> $f'(x)$ changes sign from $-$ to $+$

If $x=a$ is a local maximum point, then $f'(a)=0$ <u>and</u> $f'(x)$ changes sign from $+$ to $-$

Ex



Here $x = a$ is neither a loc. max. point nor a local min. point.
It is a terrace point.

Definition If $f'(a) = 0$ then $x = a$ is a stationary point.

Ex $f(x) = x^3 - 6x^2 + 5$. We find the stationary points.

- simply solve the eq. $f'(x) = 0$

First we find $f'(x) = 3x^2 - 6 \cdot 2x + 0$

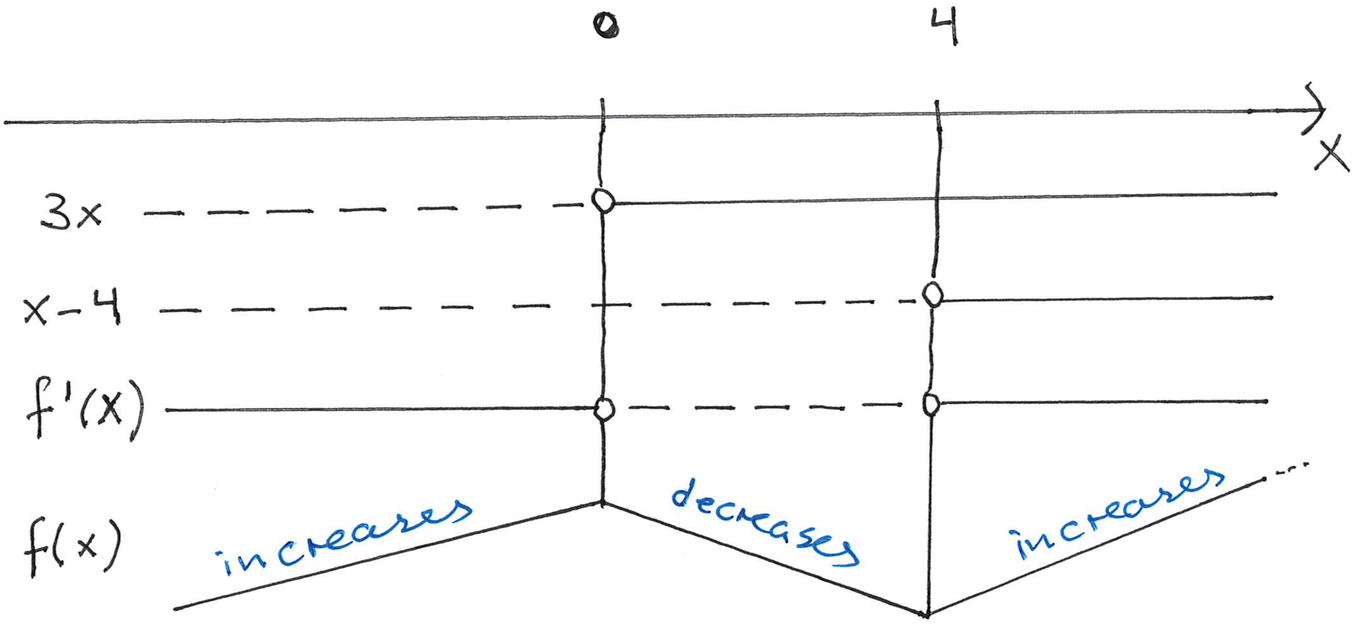
$$= 3x^2 - 12x$$

$$= x(3x - 12) = 3x(x-4)$$

So $f'(x) = 0$ has solutions $\underline{\underline{x = 0}}$ and $\underline{\underline{x = 4}}$

Where is $f(x)$ increasing/decreasing?

- we determine the sign of $f'(x)$ by a sign diagram.



$f(x)$ is strictly increasing for $x \leq 0$ (so $x \in \langle \leftarrow, 0]$)

$f(x)$ is strictly decreasing for $0 \leq x \leq 4$ (so $x \in [0, 4]$)

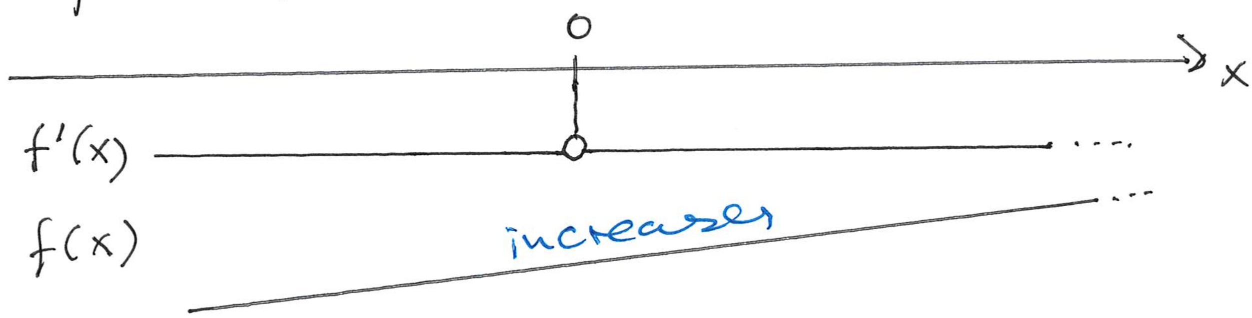
$f(x)$ is strictly increasing for $x \geq 4$ (so $x \in [4, \infty\rangle$)

Then $x = 0$ is a local maximum point

and $x = 4$ is a local minimum point

Ex $f(x) = x^3 + 1$

$f'(x) = 3x^2$, so $x = 0$ is the only stationary point for $f(x)$



Conclusion $f(x)$ is strictly increasing for all numbers on the number line ($x \in \mathbb{R}$)

Start : 11.00

2. Global max/min

> **The extreme value theorem** If $f(x)$ is a continuous function on the interval $D_f = [a, b]$ then $f(x)$ has a global maximum and a global min.

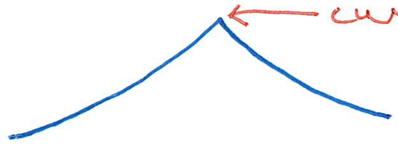
continuous function = unbroken graph (one snake)

Possible max/min points:

(*) stationary points (solve $f'(x) = 0$)

(*) cusp points (where $f'(x)$ is not defined)

cusp point



(*) end points (a and b)

Ex $f(x) = x^3 - 6x^2 + 5$ and $D_f = [-1, 7]$

Find max/min. of $f(x)$.

Solution

(*) stationary points: $f'(x) = 3x^2 - 12x = 0$ gives $\underline{x = 0}$, $\underline{x = 4}$

(*) cusp points: none

(*) end points: $\underline{x = -1}$, $\underline{x = 7}$

These four points are my candidate points for max/min.

Calculate:

$f(-1) = -2$ $\quad f(4) = -27$

$f(0) = 5$ $\quad f(7) = 54$

So $x = 4$ gives the global minimum $f(4) = \underline{\underline{-27}}$

and $x = 7$ gives the glob. maximum $f(7) = \underline{\underline{54}}$

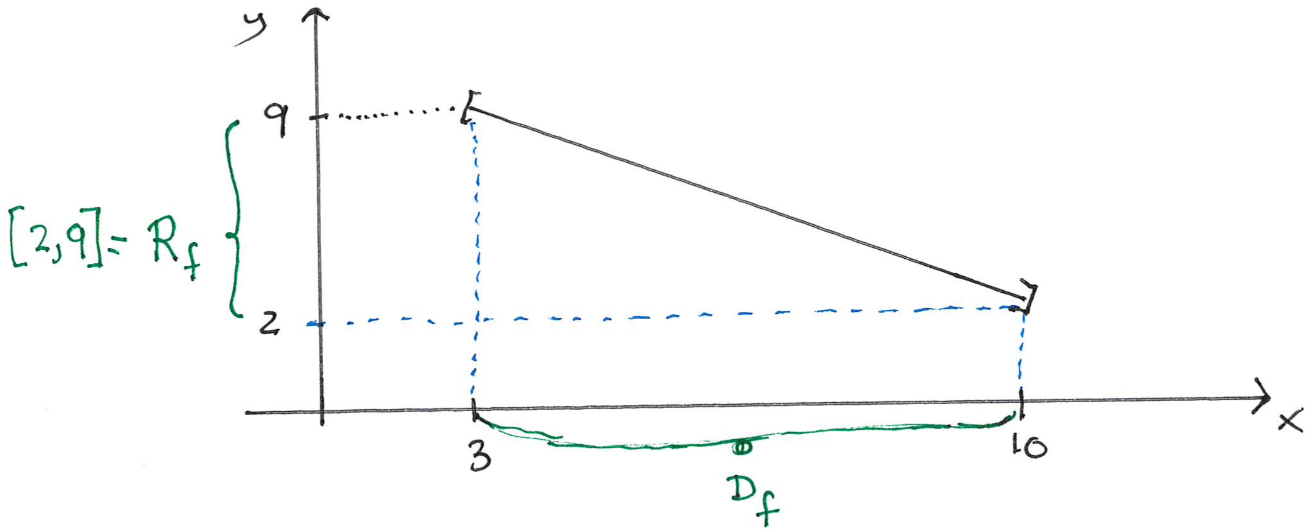
Ex $f(x) = 12 - x$ with $D_f = [3, 10]$

(*) $f'(x) = -1 \neq 0$: no stationary points

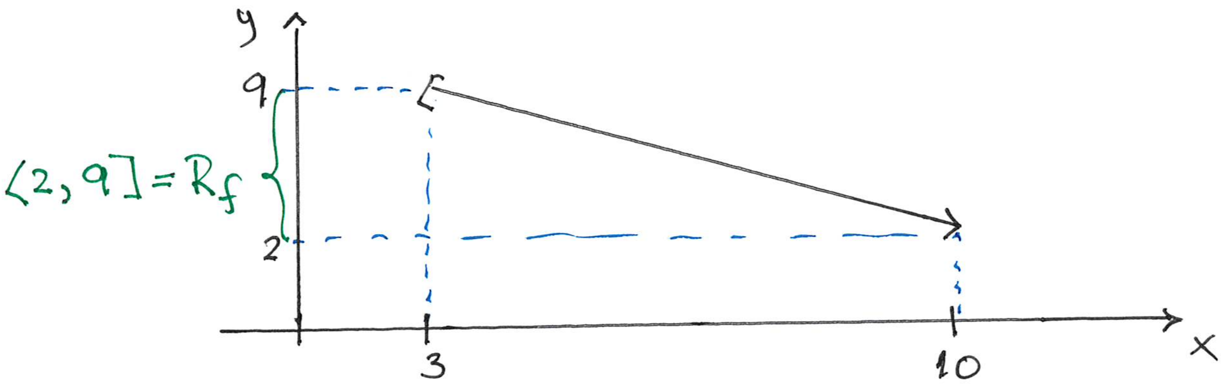
(*) no cusp points

(*) end points: $x = 3$ is a max. point
$x = 10$ is a min. point

($f(x)$ is decreasing in the whole domain)



Ex $f(x) = 12 - x$ with $D_f = [3, 10\rangle$



So $x = 3$ is still the max. point and $f(3) = 9$ is the max. value, but there is no minimum point, and no minimum value.

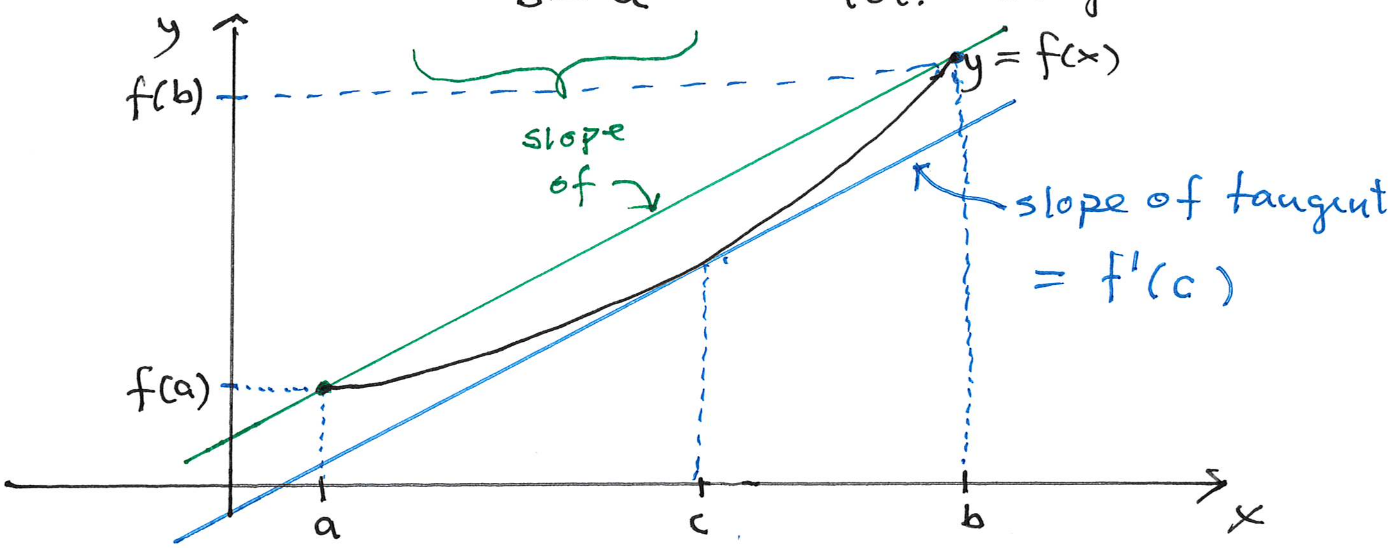
3. The mean value theorem

If $f(x)$ is continuous in the interval $[a, b]$
(connected graph)

and differentiable, then
(no cusps)

there is a number c between a and b
$(a < c < b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{\text{tot. change in } y}{\text{tot. change in } x}$$



green and blue line are parallell.

<u>Ex</u> $f(x) = e^x + x^2$. Then $f(0) = e^0 + 0^2 = 1$
and $f(1) = e^1 + 1^2 = e + 1$ (so $a = 0$, $b = 1$)

By the mean value thm. there is a number c between 0 and 1 such that $f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{e + 1 - 1}{1} = e$

<u>Note</u> $f'(x) = e^x + 2x$ (easy) but we cannot solve the eq $e^x + 2x = e$ (no exact solution)