EBA 1180, lecture 23, 22 Nov. 2022, Runar Ile

Plan 1. Repetition: l'Hôpital's rule (probl. 1h)
Cost functions (probl. 3c)

2. Elasticity

1. Rep: l'Hôpital's rule

Used for limits: $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

- Differentiate the numerator and the denominator seperately
- consider the limit of the new fraction

Probl 1h

$$\lim_{x\to 1} \frac{\ln(x)}{\sqrt{x}-1} \overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{\left(\frac{1}{2\sqrt{x}}\right)} = \frac{\left(\frac{1}{1}\right)}{\left(\frac{1}{2\sqrt{1}}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = \underline{\underline{2}}$$

"$\frac{0}{0}$"

Also

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}-1} \overset{\text{l'Hôp}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\left(\frac{1}{2\sqrt{x}}\right)} \qquad \left| \frac{\cdot x \cdot 2\sqrt{x}}{\cdot x \cdot 2\sqrt{x}} = 1 \right.$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}\cdot\sqrt{x}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = \underline{\underline{0}}$$

Meaning of $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}-1} = 0$ is that

the line $y = 0$ is a horizontal asymptote

for the function $\frac{\ln(x)}{\sqrt{x}-1}$.

Rep: Cost functions

$C(x)$ is a cost function if ($x \geq 0$ and is the number of units produced)

① $C(0) > 0$

② $C(x)$ is increasing ($C'(x) \geq 0$)

③ $C(x)$ is convex ($C''(x) \geq 0$)

Average unit cost: $A(x) = \dfrac{C(x)}{x}$

Cost optimum: the minimum point (x-value) for $A(x)$

Nice result: If $C''(x) > 0$ then the cost optimum is the solution of the equation $A(x) = C'(x)$

Probl. 3c $C(x) = 400 \cdot e^{0.001 \cdot x^2}$ ($x \geq 0$)

is a cost function because

① $C(0) = 400 \cdot e^{0.001 \cdot 0^2} = 400 \cdot e^0 = 400 > 0$

② $C'(x) \overset{\text{chain rule}}{=} 400 \cdot 0.001 \cdot 2x \cdot e^{0.001 \cdot x^2}$
$= 0.8x \cdot e^{0.001 \cdot x^2} \geq 0$ for all $x \geq 0$

③ $C''(x) \overset{\text{prod. rule}}{=} 0.8 \cdot e^{0.001 \cdot x^2} + 0.8x \cdot 0.002 \cdot x \cdot e^{0.001 \cdot x^2}$

$= \underbrace{0.8}_{>0} \cdot \underbrace{(1 + 0.002 \cdot x^2)}_{>0} \cdot \underbrace{e^{0.001 \cdot x^2}}_{>0} > 0$ for all x

So $C(x)$ is a cost function which is strictly convex.

Since $C''(x) > 0$ for all $x > 0$, the cost optimum is the solution of the eq.

$$C'(x) = A(x)$$

that is $$0.8 \cdot x\, e^{0.001 \cdot x^2} = \frac{400 \cdot e^{0.001 \cdot x^2}}{x} \quad \Big| \cdot x$$

gives $$0.8 \cdot x^2 \cdot e^{0.001 \cdot x^2} = 400 \cdot e^{0.001 \cdot x^2} \quad \Big| : \underbrace{e^{0.001 x^2}}_{> 0}$$

gives $$0.8\, x^2 = 400$$

$$x^2 = \frac{400}{0.8} = 500$$

and $$x = \underline{\underline{\sqrt{500}}} = \underline{\underline{22.36}} \qquad (x \geq 0)$$

And the minimal average unit cost is

$$A(\sqrt{500}) \overset{\text{by nice result}}{=} C'(\sqrt{500})$$

$$= 0.8 \cdot \sqrt{500} \cdot e^{0.001 \cdot 500} = \underline{\underline{29.49}}$$

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2. Elasticity

p = price/unit

$D(p)$ = demand of a commodity with price p = # sold units

The problem of comparing units. Start: 11.00

Ex A barrel of North Sea crude oil costs \$ 87.53

A litre of — " — costs NOK 5.64

The price elasticity of the demand is

$$\varepsilon = \frac{\text{relative change in demand}}{\text{relative change in price}}$$

these numbers are independent of units

<u>Ex</u> In a month the price of a commodity drops fra 12 thousand to 10 thousand and the demand increases from 50 mill. to 60 mill. Then

$$\varepsilon = \frac{\left(\frac{60-50}{50}\right)}{\left(\frac{10-12}{12}\right)} = \frac{\left(\frac{10}{50}\right)}{\left(\frac{-2}{12}\right)} = \frac{120}{-100} = \underline{\underline{-1.2}}$$

<u>Interpretation</u> If the price increases by 1 % then the demand falls by 1.2 %

<u>Theory</u>:
Suppose the price is changed from p to p+h
Then the relative change in price is

$$\frac{p+h-p}{p} = \frac{h}{p}. \quad \text{Then}$$

$$\frac{\text{relative change in demand}}{\text{relative change in price}} =$$

$$= \frac{\left(\frac{D(p+h) - D(p)}{D(p)}\right)}{\left(\frac{h}{p}\right)} \qquad \Big| \cdot \frac{p \cdot D(p)}{p \cdot D(p)} = 1$$

$$= \frac{D(p+h) - D(p)}{h} \cdot \frac{p}{D(p)}$$

$$\Big\downarrow h \to 0 \quad \text{(the price change approaches 0)}$$

$$\varepsilon(p) = D'(p) \cdot \frac{p}{D(p)}$$

This is the momentary price elasticity of the demand function.

Ex $D(p) = 50 - p$ for $0 < p < 50$

Then $D'(p) = -1$ and $\varepsilon(p) = \frac{(-1) \cdot p}{50 - p}$

$$= \frac{-p}{50 - p}$$

Important question

Is the revenue going up or down if we increase the price a little?

Revenue $R(p) = p \cdot D(p)$

- Is $R(p)$ increasing or decreasing ?

The marginal revenue w.r.t. price is

$$R'(p) \overset{\text{prod. rule}}{=} 1 \cdot D(p) + p \cdot D'(p)$$

$$= D(p) \cdot \left[1 + \frac{p \cdot D'(p)}{D(p)}\right]$$

$$= \underbrace{D(p)}_{\text{always pos.}} \cdot \underbrace{\left[1 + \varepsilon(p)\right]}_{\text{positive or negative ?}}$$

If $\varepsilon(p) < -1$

we get neg. $R'(p)$

so $R(p)$ is decreasing

Say: elastic demand

If $\varepsilon(p) > -1$

we get pos. $R'(p)$

so $R(p)$ is increasing

Say: inelastic demand

If $\varepsilon(p) = -1$

the demand is unit elastic

Ex $D(p) = 50 - p$ and got $\varepsilon(p) = \dfrac{-p}{50-p}$

Q: In what price range do we have elastic demand ?

Solve inequality : $\varepsilon(p) < -1$

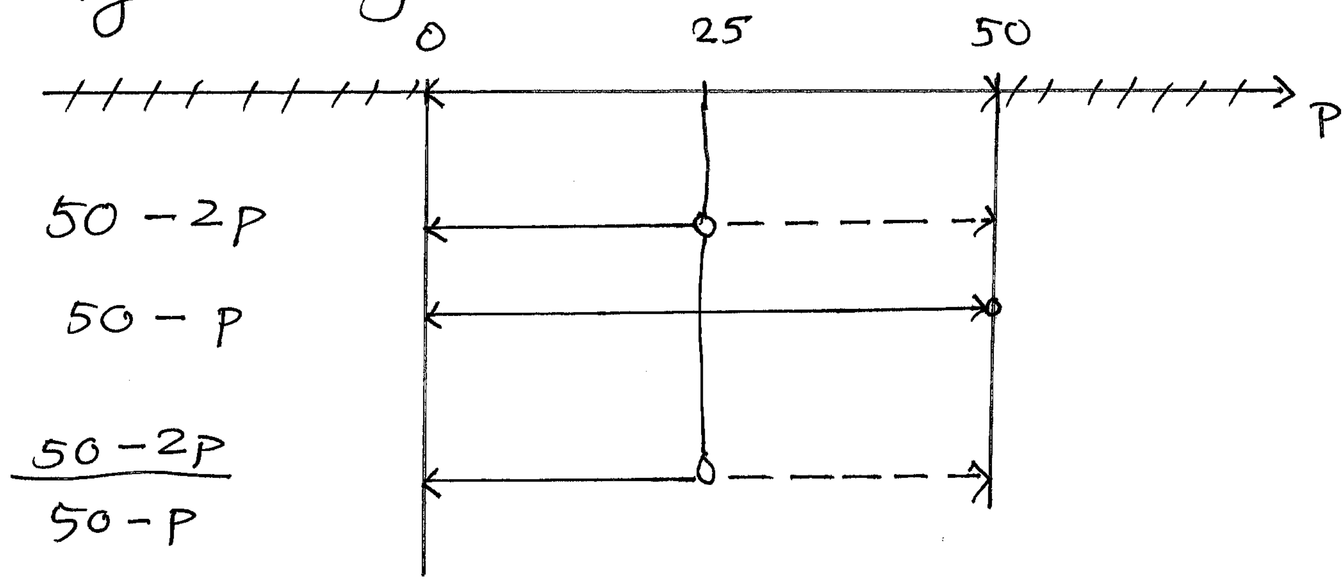
so $\dfrac{-p}{50-p} < -1 \quad \Big| +1$

$$\frac{-p}{50-p} + 1 < 0$$

so $$\frac{-p + 50 - p}{50-p} < 0$$

So $$\frac{50-2p}{50-p} < 0$$

sign diagram:



so elastic demand w.r.t. price
for p in $\langle 25, 50 \rangle$

and inelastic demand for p in $\langle 0, 25 \rangle$

and unit elastic demand for $p = 25$.