EBA1180, lecture 24, 23 Nov. 2022, Runar Ile

Plan
1. Repetition: Elasticity
2. Linear approximation
3. Higher degree Taylor polynomials
4. About the exam.
5. How to prepare for the exam.

1. Rep.: Elasticity

Ex: p = price/unit. Demand function $D(p) = 200 \cdot e^{-0.01p}$

Calculate $\varepsilon(p)$ - the elasticity function.

$$D'(p) = -0.01 \cdot 200 \cdot e^{-0.01p} = -2e^{-0.01p}$$

so
$$\varepsilon(p) = \frac{D'(p) \cdot p}{D(p)} = \frac{-2\cancel{e^{-0.01p}} \cdot p}{200 \cdot \cancel{e^{-0.01p}}} = \underline{\underline{-0.01p}}$$

The demand is elastic w.r.t. price if

$\varepsilon(p) < -1$ that is $-0.01p < -1$ (an inequality)

that is $\underline{\underline{p > 100}}$

Meaning: If $p > 100$ then a small increase in price gives a decline in revenue.

Ex: $\varepsilon(110) = -1.1$, so a price increase of 1 % from 110 gives a demand decrease of 1.1 %

The demand is inelastic w.r.t. price if

$\varepsilon(p) > -1$, that is $-0.01p > -1$ so

$\underline{\underline{p < 100}}$

Meaning: If $p < 100$, a small price increase gives an increase in revenue.

Ex: $\varepsilon(80) = -0.8$, so 1% price increase gives 0.8% demand decrease.

If $\varepsilon(p) = -1$ (so $p=100$) then demand is unit elastic w.r.t. price

Meaning: No (or very little) change in revenue if price is changed a little from 100.

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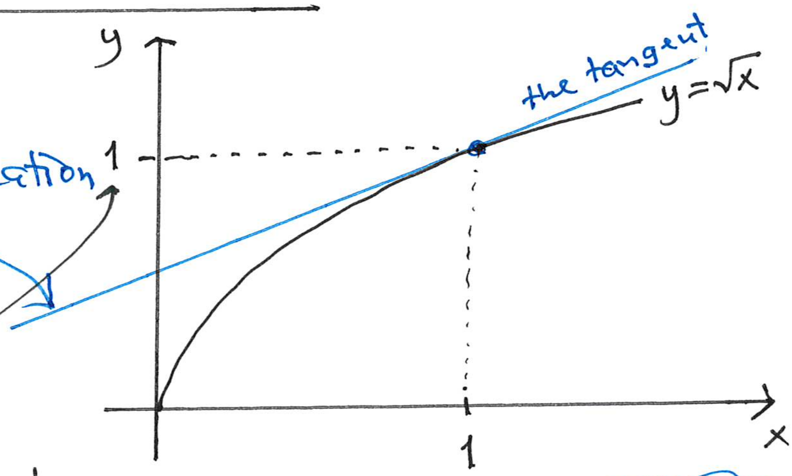
2. Linear approximation

Ex: $f(x) = \sqrt{x}$



The linear approximation of $f(x)$ about $x=1$

We can the find the expression of the tangent line by the point-slope formula

$$y - 1 = f'(1)(x-1)$$

so $y - 1 = \frac{1}{2}(x-1)$

or $y = \underbrace{\overset{=f(1)}{1} + \overset{=f'(1)}{\tfrac{1}{2}}(x-1)} \overset{\text{notation}}{=} P_1(x)$

– is called the deg. one Taylor polynomial of $\sqrt{x}$ about $x=1$.

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$= \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{x^{1/2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$f'(1) = \frac{1}{2\cdot\sqrt{1}} = \frac{1}{2}$$

Ex $P_1(1.1) = 1 + \frac{1}{2}(1.1-1) = 1.05$

(check: $\sqrt{1.1} = 1.04881\ldots$)

3. Higher degree Taylor polynomials

Ex $f(x) = \sqrt{x}$

The Taylor polynomial of degree 2 for $\sqrt{x}$ about $x = 1$ is

$$P_2(x) = \overbrace{f(1) + f'(1)\cdot(x-1)}^{P_1(x)} + \frac{f''(1)}{2}(x-1)^2$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$$

$f'(x) = \frac{1}{2}\cdot x^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot x^{-\frac{1}{2}-1}$

$= -\frac{1}{4}\cdot\frac{1}{x^{3/2}}$

$= -\frac{1}{4x\sqrt{x}}$

$f''(1) = -\frac{1}{4\cdot 1\sqrt{1}}$

$= -\frac{1}{4}$

Pattern

$$\boxed{P_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2}$$

- So $a = 1$ in the example

$$\sqrt{2} = f(2) \approx P_2(2) = 1 + \frac{1}{2}(2-1) - \frac{1}{8}(2-1)^2$$

$$= 1 + \frac{1}{2} - \frac{1}{8} = 1.375$$

(check $\sqrt{2} = 1.41421\ldots$)

$$P_2(1.2) = 1 + \frac{1}{2}(1.2-1) - \frac{1}{8}(1.2-1)^2$$

$$= 1 + 0.1 - 0.005 = 1.0950$$

(check $\sqrt{1.2} = 1.0954\ldots$)

Start 11.00

Ex $f(x) = \sqrt{x}$ about $x = 1$

Then the third degree Taylor polynomial for $f(x)$ about $x = 1$ is:

$$P_3(x) = \underbrace{P_2(x)}_{\text{have already done this!}} + \frac{f'''(1)}{6}(x-1)^3$$

$$f''(x) = -\frac{1}{4x^{3/2}} = -\frac{1}{4}\cdot x^{-3/2}$$

$$f'''(x) = -\frac{1}{4}\cdot\left(-\frac{3}{2}\right)\cdot x^{-\frac{5}{2}} = \frac{3}{8x^2\sqrt{x}}$$

$$f'''(1) = \frac{3}{8}$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{\left(\frac{3}{8}\right)}{6}(x-1)^3$$

$$= 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$$

$$P_3(1.2) = 1 + \frac{1}{2}(1.2-1) - \frac{1}{8}(1.2-1)^2 + \frac{1}{16}(1.2-1)^3$$

$$= 1.0955$$

Pattern (3rd degree Taylor polynomial)

$$P_3(x) = \underbrace{\overbrace{f(a) + f'(a)(x-a)}^{P_1(x)} + \frac{f''(a)}{2}(x-a)^2}_{P_2(x)} + \frac{f'''(a)}{6}(x-a)^3$$

The degree n Taylor polynomial for $f(x)$ about $x=a$ is:

$$\boxed{\begin{array}{l} P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n \\ \text{where } n! = n\cdot(n-1)\cdot(n-2)\cdot\ \dots\ \cdot 3\cdot 2\cdot 1 \end{array}}$$

4. About the exam

- 12 problems of equal weight
  - might have subproblems
- 3 hours (14–17), find out where!
- The exam paper is in Wiseflow
  - need to bring your computer with the Flowlock browser installed (before the exam!!)
- you write your answers on paper
- I am grading
- All the problems should be (very) recognizable from the tutoring and the lectures
- Rather basic and central problems in the first half (at least)
- The problems are <u>not</u> ordered according to the lecture plan.
- Support materials: BI-calculator. Ruler.
- The exam counts for 20% of final grade.

5. How to prepare

① Relevant material:
- lec. notes
- tutoring probl.
- earlier multiple choice exams
- also text book

② Try to „solve" the problems in your head!
- what is the plan (in detail)
- what kind of knowledge is required?
- what kind of obstacles may occur?

③ If I get a wrong answer:
- what went wrong? - the plan?
  - the calculations?

④ When you have solved a problem - what did you learn?

⑤ Learn the basics well!
- definitions, concepts ("words")

⑥ Basic problems are the most important.

Ex: $e^x = 5$ or $\ln(x+5) = 0$