EBA 1180, lecture 3, 31 Aug. 2022, Runar Ile

Plan: 1. A few examples
2. Total present value of a cash flow

1. A few examples

Problem The value of Kåre's flat increases by 10% the first year and decreases by 30% the second year. Compute the relative change for the two years combined.

Solution

Relative change for the first year : $r_1 = 0.1$

—— " —— second year : $r_2 = -0.3$

Rate of change for the first year : $1 + r_1 = 1.1$

—— " —— second year : $1 + r_2 = 0.7$

—— " —— for the two years combined :

$$(1+r_1)\cdot(1+r_2) = 1.1 \cdot 0.7 = 0.77$$

So the relative change for the two years is $0.77 - 1 = -0.23 = \underline{\underline{-23\%}}$

Pattern Relative changes of value : $r_1, r_2, \ldots, r_n$ gives combined relative change

$$\underbrace{(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_n)}_{\text{combined rate of change}} - 1$$

EX Deposit (pricipal) $B_0 = 50\,000$
Interest $r = 4\%$ (annual compounding)

After 5 years the balance is

$$50\,000 \cdot (1+4\%)^5 = 60\,832.65$$

Calculator: 50000 $\boxed{\times}$ 1.04 $\boxed{y^x}$ 5 $\boxed{=}$

Problem Deposit 50 000
Nominal interest 4 %
Monthly compounding

a) Determine the balance after 5 years.

b) Determine the effective interest.

Solution a) After 5 years the balance is

$$50\,000 \cdot \left(1+\frac{4\%}{12}\right)^{12\cdot 5}$$

$$= 50\,000 \cdot \left(1+\frac{0.04}{12}\right)^{60} = \underline{\underline{61\,049.83}}$$

b) Effective interest r_{eff} = the annual interest which gives the same balance as the periode rate

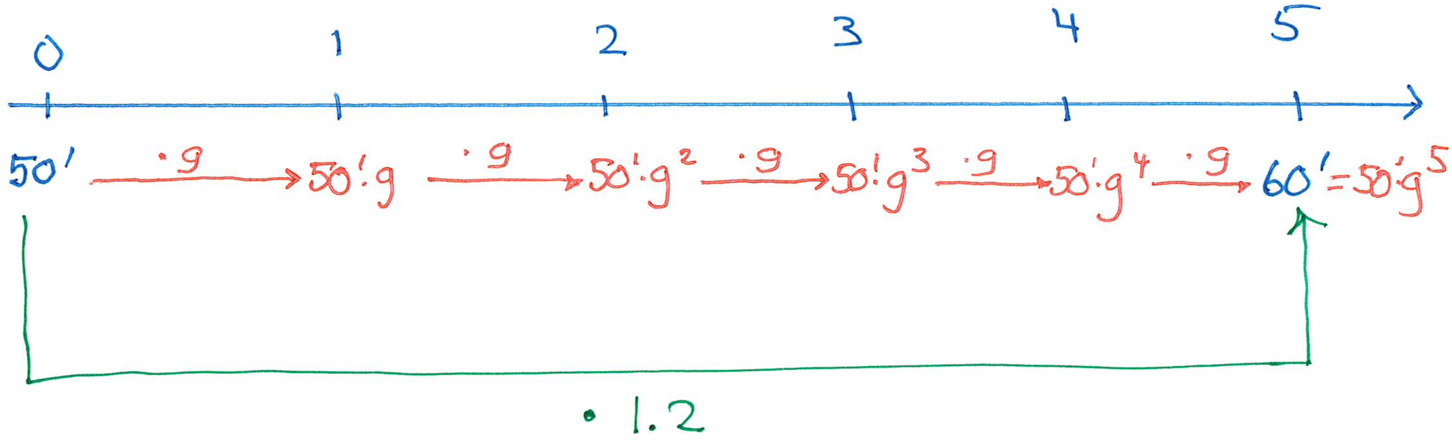
In (a) $1 + r_{eff} = \left(1+\frac{4\%}{12}\right)^{12} = 1.040742$

so $r_{eff} = 4.0742\%$

Problem After 5 years of added interest the deposit of 50000 has become 60000. Calculate the effective interest.

Solution The 5-year growth factor is

$$1 + \frac{60000 - 50000}{50000} = 1.2$$



Let g be the annual growth factor.

Then $50000 \cdot g^5 = 60000$

or $g^5 = 1.2$

$$g = (g^5)^{\frac{1}{5}} = 1.2^{\frac{1}{5}} \left(= \sqrt[5]{1.2}\right)$$

$$= 1.2^{0.2} = 1.03714$$

So $\underline{\underline{r_{eff} = 3.714\,\%}}$

2. Total present value of a cash flow

Present value of an amount (K) paid n years from now with interest r

= what you have to deposit today (K_0) for the balance to be K n years from now with interest r.

Since $K = K_0 \cdot (1+r)^n$ we have

$$K_0 = \frac{K}{(1+r)^n}$$

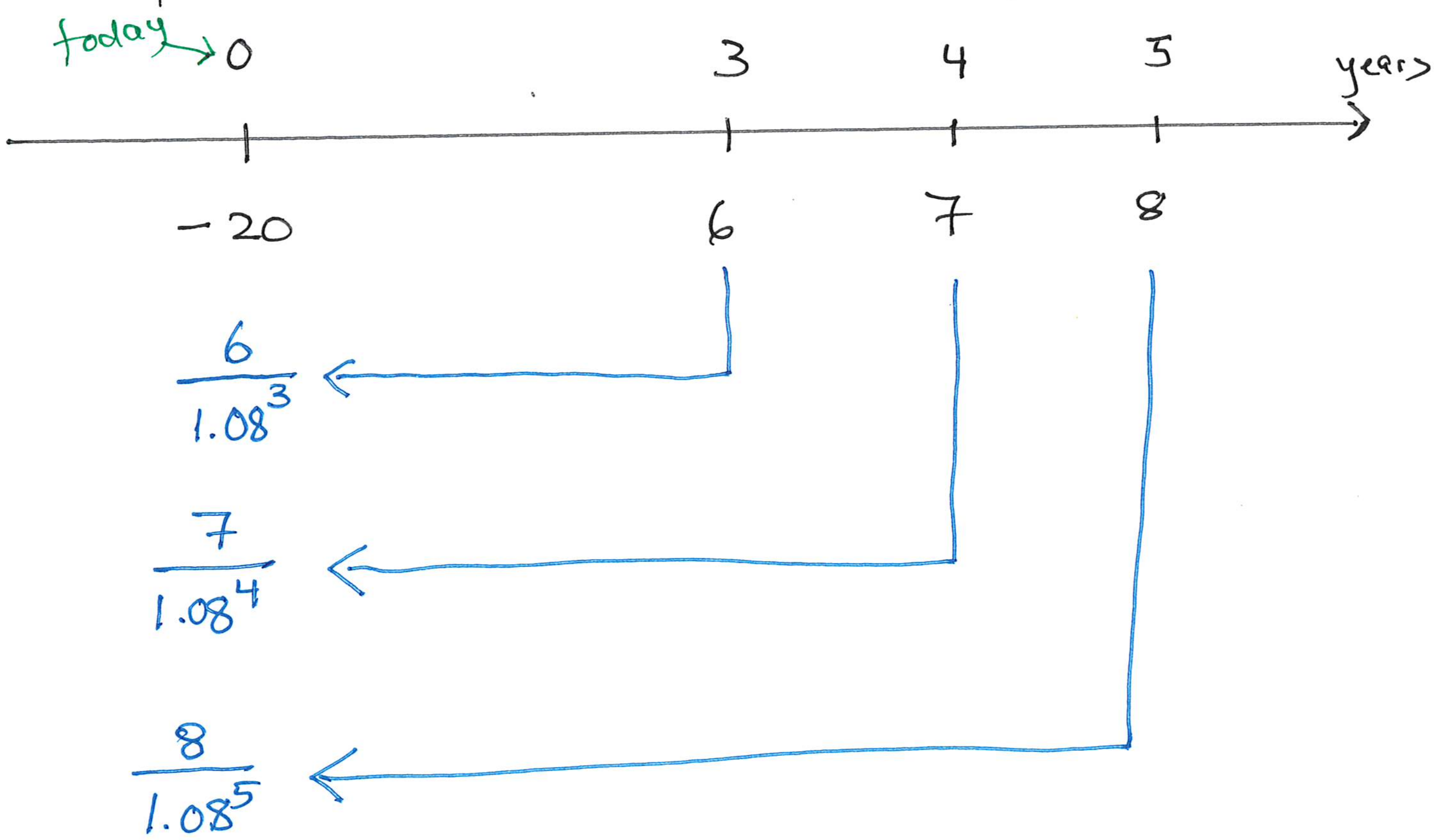
Ex 50 000 (K) 3 years from now with 4% interest has present value

$$K_0 = \frac{50\,000}{1.04^3} = \underline{\underline{44\,449.82}}$$

We can extend this to a cash flow (several payments combined)

Ex You pay 20 mill. today, and get back
6 mill. after 3 years
7 mill. after 4 years
8 mill. after 5 years

With 8 % interest, the present value of the cash flow is the sum of the present values of each of the payments



The sum = the total present value of the cash flow

$$= -20 + \frac{6}{1.08^3} + \frac{7}{1.08^4} + \frac{8}{1.08^5} = \underline{\underline{-4.65}}$$

(bad investment)

The <u>internal rate of return</u> (IRR) is the interest which makes the total present value of the cash flow equal to 0.

In general hard to calculate the IRR by hand. We have to solve the equation

$$f(x) = -20 + \frac{6}{(1+x)^3} + \frac{7}{(1+x)^4} + \frac{8}{(1+x)^5} = 0$$

(Answer: $x \approx 1.12\%$)