EBA1180, lecture 6, 13 Sept. 2022, Runar Ile

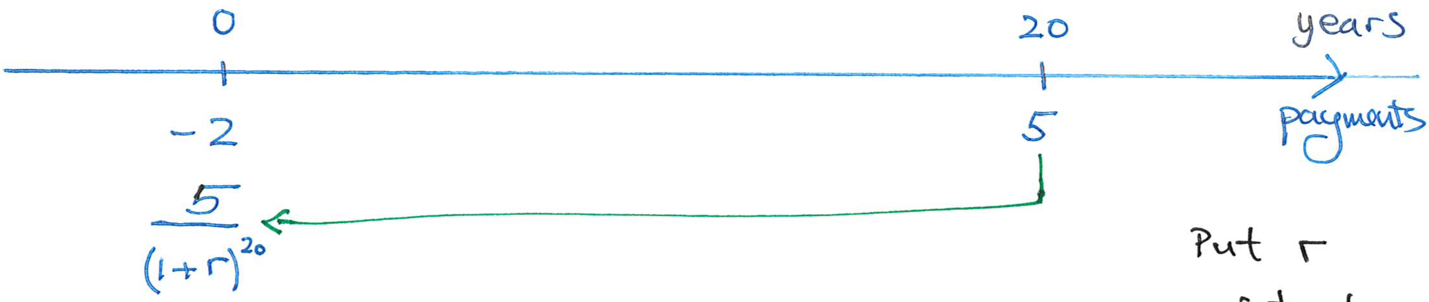
Plan: 1. Repetition

2. Linear and quadratic equations

3. Equations with parametres: the abc-formula

1. Repetition

Probl. 5 (last week) You consider investing 2 mill. today to receive 5 mill 20 years from now.



Put r = int. rate of return (IRR)

a) Annual compounding gives equation (tot. pres. value = 0):

$$-2 + \frac{5}{(1+r)^{20}} = 0$$

so
$$\frac{5}{(1+r)^{20}} = 2 \qquad \Big| \cdot (1+r)^{20}$$

$$5 = 2\cdot(1+r)^{20} \qquad \Big| : 2$$

get
$$(1+r)^{20} = \frac{5}{2}$$

so
$$1+r = \sqrt[20]{\frac{5}{2}} = \left(\frac{5}{2}\right)^{\frac{1}{20}}$$

and
$$r = \left(\frac{5}{2}\right)^{\frac{1}{20}} - 1 = 4.69\,\%$$

2.5 $\boxed{y^x}$ 20 $\boxed{1/x}$ $\boxed{-}$ 1 $\boxed{=}$

b) With quarterly compounding, the quarterly IRR r gives the equation

$$\frac{5}{(1+r)^{80}} = 2$$ (80 interest periods $= 4 \cdot 20$)

We get $r = \left(\frac{5}{2}\right)^{\frac{1}{80}} - 1 = 1.152\%$

which gives the nominal annual IRR as $4 \cdot 1.152\% = \underline{\underline{4.61\%}}$

c) With monthly compounding the monthly IRR is

$$r = \left(\frac{5}{2}\right)^{\frac{1}{240}} - 1 = 0.3825\%$$

and nominal annual IRR is then $12 \cdot 0.3825\% = \underline{\underline{4,59\%}}$

d) With continuous compounding the annual growth factor is e^r

so the tot. pres. value of the cash flow is $-2 + \frac{5}{(e^r)^{20}}$ which is supposed to be 0 so eq. $\frac{5}{(e^r)^{20}} = 2$ $\left| \cdot \frac{(e^r)^{20}}{2} \right.$

which gives $(e^r)^{20} = \frac{5}{2}$

so $e^r = \left(\frac{5}{2}\right)^{\frac{1}{20}} = 1.0469$

Have to try different values of r

A good answer is $r = \underline{4.58\%}$

(or $r = \frac{\ln 5 - \ln 2}{20}$)

Problem You deposit 2 mill today, annual (nominal) interest is 12 % with continuous compounding. Determine the balance after 1 year and 7 months.

Solution $2 \text{ mill} \cdot \underbrace{e^{0.12}}_{1 \text{ year}} \cdot \underbrace{(e^{0.12})^{\frac{7}{12}}}_{7 \text{ mnths.}}$

$= 2 \text{ mill} \cdot e^{0.12 + 0.12 \cdot \frac{7}{12}} = 2 \text{ mill} \cdot e^{0.12\left(1+\frac{7}{12}\right)}$

$= 2 \text{ mill} \cdot e^{0.\cancel{12} \cdot \frac{19}{\cancel{12}}}$

$= 2 \text{mill} \cdot e^{0.19} = \underline{2{,}4185 \text{ mill}}.$

Start : 11.10

2. Linear and quadratic equations

A linear expression $ax+b$ (a and b are numbers and $a \neq 0$)

Ex $4x-3$ $(a=4,\ b=-3)$

A linear equation An eq. which can be transformed into an equivalent equation of the form $ax+b=0$ $(a \neq 0)$

Ex The eq. $\frac{1}{x+3} = \frac{2}{x+4}$ $\quad | \cdot (x+3)(x+4)$

Multiply with a common denominator on each side.

is transformed to $x+4 = 2\cdot(x+3)$

We use the distributive law on the RHS.

$$x+4 = 2x+6$$

Subtract $2x+6$ on each side

$$-x-2=0 \quad (a=-1,\ b=-2)$$

$$(x \neq -3,\quad x \neq -4).$$

A quadratic expression ax^2+bx+c

a, b, c are numbers and $a \neq 0$

quad. eq. – eq. which can be made into an equivalent eq: $ax^2+bx+c=0$

Ex $\frac{1}{x} + \frac{2}{(x+1)} = 3 \quad \Big| \cdot x(x+1)$

$$x+1 + 2x = 3x(x+1)$$

$$3x+1 = 3x^2 + 3x$$

Subtract $3x + 1$ on each side

$$3x^2 - 1 = 0 \qquad (a=3,\ b=0,\ c=-1)$$

$(x \neq 0,\ x \neq -1)$

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3. Equations with parametres: the abc-formula

If $a \neq 0$ then $ax^2 + bx + c = 0$

has solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex $3x^2 + 4x - 5 = 0 \qquad (a=3,\ b=4,\ c=-5)$

Then $x = \dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$

$$= \frac{-4 \pm \sqrt{16+60}}{6} = \frac{-4 \pm \sqrt{4 \cdot 19}}{6} =$$

$$= \frac{-4 \pm \sqrt{4} \cdot \sqrt{19}}{6} = \frac{\cancel{2}(-2 \pm \sqrt{19})}{\cancel{2} \cdot 3} = \underline{\underline{\frac{-2 \pm \sqrt{19}}{3}}}$$

$$= \underline{\underline{-\frac{2}{3} \pm \frac{\sqrt{19}}{3}}}$$