EBA1180, lecture 7, 14 Sept. 2022, Runar Ile

Plan: 1. Quadratic equations
2. Completing the square
3. Equations with given solutions

1. Quadratic equations

- an eq. which can be transformed into the standard form $ax^2 + bx + c = 0$ $(a \neq 0)$

It has the solution(s):

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Three cases:

$b^2 - 4ac > 0$ gives two solutions

$b^2 - 4ac = 0$ gives one solution

$b^2 - 4ac < 0$ gives no solutions

Problem Determine the number of solutions:

a) $x^2 + 5x + 6 = 0$ $5^2 - 4 \cdot 1 \cdot 6 = 1 > 0$: two solutions

b) $-x^2 + 2x - 1 = 0$ $2^2 - 4 \cdot (-1)(-1) = 0$: one solution

c) $4x^2 - 5x - 5 = 0$ $(-5)^2 - 4 \cdot 4 \cdot (-5) > 0$: two solutions

The quadratic formula is often inefficient:

Ex $-3x^2+7=0$ $(a=-3,\ b=0,\ c=7)$

$$-3x^2=-7 \quad | : -3$$

$$x^2=\frac{7}{3}$$

$|x| = \sqrt{x^2} = \sqrt{\frac{7}{3}}$ so $x = \pm\sqrt{\frac{7}{3}}$

Ex $2x^2-6x=0$ $(a=2,\ b=-6,\ c=0)$

$$2(x^2-3x)=0 \quad | : 2$$

$$x^2-3x=0$$

$x(x-3)=0$ then

either $x=0$ or $x-3=0$

$x=3$

Pattern: If $a \cdot b = 0$ then $a=0$ or $b=0$ (or both)

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2. Completing the square

Ex $x^2+6x-16=0$

Claim: $x^2+6x=(x+3)^2-9$

because $(x+3)^2 = x^2+2\cdot3\cdot x+3^2$
$= x^2+6x+9$

$+6:2$

$$(x+3)^2-9-16=0$$

$$(x+3)^2=25$$

so $x+3=5$ or $x+3=-5$

$x=2$ $x=-8$

Problem Solve the quadratic eq. by completing the square.

a) $x^2 - 8x - 33 = 0$

Solution: $\frac{-8}{2} = -4$ so $x^2 - 8x = (x-4)^2 - 4^2$

(because $(x-4)^2 = x^2 - 2\cdot 4\cdot x + (-4)^2 = x^2 - 8x + 16$

Rewrite eq: $(x-4)^2 - 16 - 33 = 0$

$(x-4)^2 = 33 + 16 = 49$

so $x - 4 = 7$ or $x - 4 = -7$

$\underline{\underline{x = 11}}$ $\underline{\underline{x = -3}}$

b) $x^2 + 2x = 63$

Solution $x^2 + 2x = (x+1)^2 - 1^2$ so

rewrite eq: $(x+1)^2 - 1 = 63$

so $(x+1)^2 = 64$

so $x + 1 = 8$ or $x + 1 = -8$

$\underline{\underline{x = 7}}$ $\underline{\underline{x = -9}}$

Start: 11.00

3. Equations with given solutions

Problem Solve the equation

$$(x-4)(x+5)=0$$

Solution If a product of two numbers is equal to zero: $a \cdot b = 0$

then at least one of the numbers has to be zero: $a=0$ or $b=0$

so when $(x-4)\cdot(x+5)=0$ then

$x-4=0$ or $x+5=0$

$\underline{x=4}$ $\qquad$ $\underline{x=-5}$

Problem Determine the quadratic expression x^2+bx+c with the given roots (zeros)

a) 1 and 2.

Solution: $(x-1)\cdot(x-2) = \underline{x^2-3x+2}$

b) 11 and -3.

solution: $(x-11)(x+3) = \underline{x^2-8x-33}$

Note: $3(x-1)(x-2) = 3x^2-9x+6$ has the same roots (1 and 2)

If r_1 and r_2 are solutions ('roots') to the quadratic equation

$$x^2+bx+c=0$$

then $(x-r_1)(x-r_2) = x^2 - r_2x - r_1x + (-r_1)(-r_2)$

$$= x^2-(r_1+r_2)x+r_1r_2$$

so $b=-(r_1+r_2)$ and $c=r_1r_2$

<u>Ex</u> $x^2+6x-16 = (x-2)(x+8)$ $\quad r_1=+2$, $r_2=-8$

<u>Problem</u> Solve the eq:

$$(x^2+1)(12+3x)(9-x^2)(x^2-3x+2)=0$$

A product equal to zero: one of the factors has to be zero.

$x^2+1=0$ — no solutions

or $12+3x=0$ $\quad 3(4+x)=0$ $\quad$ so $\underline{x=-4}$

or $9-x^2=0$ $\quad (3-x)(3+x)=0$ $\quad$ so $\underline{x=\pm3}$

or $x^2-3x+2=0$ $\quad (x-1)(x-2)=0$ $\quad$ so $\underline{x=1}$ or $\underline{x=2}$

<u>Problem</u> solve the eq. $x^4-12x^3+11x^2=0$

$x^2(x^2-12x+11)=0$ so

$x^2=0$ or $x^2-12x+11=0$

$(x-1)(x-11)=0$

$\underline{x=0}$ $\quad$ $\underline{x=1}$ or $\underline{x=11}$