EBA1180, lecture 8, 20 Sept. 2022, Runar Ile

Plan
1. Repetition (problems from last week)
2. Polynomial division and factorisation

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1. Repetition (problems from last week)

2m) Solve the eq. $9x^2 - 6x + 1 = 0 \quad | :9$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = 0$$

Complete the square: $\left(x - \frac{1}{3}\right)^2 \underbrace{- \left(\frac{1}{3}\right)^2 + \frac{1}{9}}_{0} = 0$

so $\left(x - \frac{1}{3}\right)^2 = 0$ so $x - \frac{1}{3} = 0$

so $\underline{\underline{x = \frac{1}{3}}}$

Alternative: Put $u = 3x$

so $u^2 = (3x)^2 = 3^2 \cdot x^2 = 9x^2$

and $-6x = -2 \cdot (3x) = -2u$. So the eq.

becomes $u^2 - 2u + 1 = 0$

so $(u-1)^2 = 0$

so $u = 1$

so $3x = 1$

so $\underline{\underline{x = \frac{1}{3}}}$

3e) Determine the quadratic eq. with the given solutions:

$x = 3 \pm \sqrt{5}$, that is $x = 3 + \sqrt{5}$, $x = 3 - \sqrt{5}$

Then $(x-(3+\sqrt{5}))(x-(3-\sqrt{5}))$ $\quad 3^2-(\sqrt{5})^2$

$= x^2-(3-\cancel{\sqrt{5}})x-(3+\cancel{\sqrt{5}})x+\overbrace{(3+\sqrt{5})(3-\sqrt{5})}$

$= x^2-6x+4$ so $\underline{\underline{x^2-6x+4=0}}$

has the given solutions $(b=-6,\ c=4)$

5c) Determine k such that

$\frac{1}{k}x^2-14x=12$ has exactly one solution.

Note: $k\neq 0$. Multiply BS with k:

$x^2-14kx=12k$

Complete the sq: $\quad (x-7k)^2=12k+(7k)^2$

has exactly one solution

if and only if the RHS $=0$, that is

$12k+49k^2=0$

i.e. $k(12+49k)=0$ i.e. $k=0$ or $12+49k=0$

– not allowed! $\quad \underline{\underline{k=-\frac{12}{49}}}$

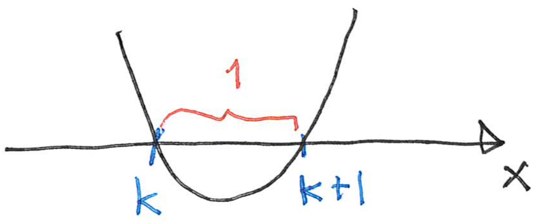
<u>Parametres</u>: numbers without explicit values

– used to describe many situations simultaneously.

<u>Ex</u> The price of a product is p kroner

Probl. 7a All polynomials x^2+bx+c which have two zeros of distance 1 from each other

can be written as

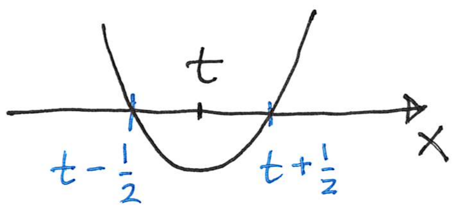


$$\overbrace{(x-k)}^{\text{zero: } x=k}\cdot\underbrace{(x-(k+1))}_{\text{zero: } x=k+1}$$

where k is the smallest zero (root)

Then $(x-k)(x-(k+1)) = \underline{\underline{x^2-(2k+1)x+k(k+1)}}$

Or



Get

$$(x-(t-\tfrac{1}{2}))(x-(t+\tfrac{1}{2}))$$

$$= \underline{\underline{x^2-2tx+t^2-\tfrac{1}{4}}}$$

Infinitly many correct solutions.

8a) Solve the eq. $(2x-\sqrt{3})\cdot(x^2-20x+99)=0$

$a \cdot b = 0$

then $a=0$ or $b=0$

so either $2x-\sqrt{3}=0$ or $x^2-20x+99=0$

$x = \underline{\underline{\dfrac{\sqrt{3}}{2}}}$

$(x-10)^2 = 10^2-99=1$

$x-10=1$ or $x-10=-1$

$\underline{\underline{x=11}}$ or $\underline{\underline{x=9}}$

2. Polynomial division and factorization

Want to divide a polynomial $f(x)$ with a polynomial $g(x)$ with a remainder $r(x)$.

$$g(x)\cdot \Big| \quad \frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{with } \deg(r(x)) < \deg(g(x))$$

gives $f(x) = q(x)\cdot g(x) + r(x)$

<u>Ex</u> $f(x) = 3x^2+2x+1$ and $g(x) = x-2$

$$\left(\boxed{3x^2}+2x+1\right) : \left(\boxed{x}-2\right) = \underset{\substack{3x^2:x\\ \parallel}}{3x} + \underset{\substack{8x:x\\ \parallel}}{8} + \frac{17}{x-2}$$

$$\begin{array}{l} -\left(3x^2-6x\right) \qquad \cdot(x-2) \\ \hline \quad \boxed{8x}+1 \\ \quad -\left(8x-16\right) \qquad \cdot(x-2) \\ \hline \qquad 17 \end{array}$$

17 is called the remainder

so $q(x) = 3x+8$ and $r(x) = 17$

<u>Can check</u>: $\left(3x+8+\frac{17}{x-2}\right)\cdot(x-2)$

$$= (3x+8)(x-2) + \frac{17}{\cancel{x-2}}\cdot\cancel{(x-2)}$$

$$= 3x^2-6x+8x-16+17 = 3x^2+2x+1 = f(x)$$

– so ok!

Two applications of polynomial division

(A) To find asymptotes of rational functions

<u>Ex</u> $$\frac{3x^2+2x+1}{x-2} = 3x+8+\frac{17}{x-2}$$

has a vertical asymptote : the line $x=2$

and a non-vertical asymptote : the line $y=3x+8$

(B) To factorise a polynomial as a product of degree 1 (linear) polynomials

<u>Ex</u> Factorise $x^3-4x^2-11x+30$ into linear factors.

<u>Solution</u> Three steps.

<u>Step I</u> Guess an integer root (zero)
[Note : Has to divide 30]

I try $\underline{x=-3}$ and get

$$(-3)^3-4\cdot(-3)^2-11\cdot(-3)+30$$
$$=-27-36+33+30 = 0$$

Then $(x-(-3)) = (x+3)$ is a factor

<u>Step II</u> Use polynomial division to find a polynomial of lower degree :

$$x^3-4x^2-11x+30 : (x+3) \overset{\text{by poly. div!}}{=} x^2-7x+10$$

Note : Remainder is 0 !

Step III We find the roots of $x^2-7x+10$

They are $x=2$, $x=5$

so $x^2-7x+10=(x-2)(x-5)$

Then $x^3-4x^2-11x+30=\underline{\underline{(x-2)(x-5)(x+3)}}$

Note 1 Not always possible to factorise!

Ex x^2+5 has no roots!

x^2+2x+3 —— " —— $\left(b^2-4ac = 2^2-4\cdot1\cdot3<0\right)$

Note 2 It can be difficult to guess roots – and the roots don't have to be integers.