

EBA1180 Mathematics for Data Science

autumn 2022

Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

R. Lucas

Lecture 1 – Basic algebra (Sec. 2.1-6, 2.8)

Here are recommended exercises from the textbook [SHSC].

Section 2.1 exercise 1

Section 2.2 exercise 1-9

Section 2.3 exercise 1-7, 9-11

Section 2.4 exercise 1-8

Section 2.5 exercise 1, 3, 3-7, 9-11

Section 2.7 exercise 1-4

Problems for the exercise session Wednesday 24 Aug. at 12-15+ in B2-065

Problem 1 Simplify the fractions as much as possible.

a) $\frac{36}{120}$ b) $\frac{60}{48}$ c) $\frac{96}{69}$ d) $\frac{112}{12}$ e) $\frac{64}{1024}$ f) $\frac{91}{26}$

g) $\frac{4}{5} \cdot \frac{2}{3}$ h) $\frac{4}{15} \cdot \frac{3}{7}$ i) $\frac{18}{4} \cdot \frac{\frac{2}{3}}{12}$ j) $\frac{\frac{2}{3}}{3}$ k) $\frac{\frac{2}{3}}{\frac{2}{3}}$ l) $\frac{\frac{2}{3}}{\frac{3}{2}}$

Problem 2 Simplify the fractions as much as possible.

a) $\frac{x^2y^2}{xy^3z}$ b) $\frac{15a(b+6)}{9(b+6)c}$ c) $\frac{x(x+1)}{(x+1)(x+2)}$ d) $\frac{x(x^2+1)}{x^2(x+1)}$
e) $\frac{(x+y)^2}{3x+3y}$ f) $\frac{(x+y)^3}{x^2+2xy+y^2}$ g) $\frac{x(x+5)}{2x^2+10x}$ h) $\frac{x^2-5}{(x-\sqrt{5})(x+\sqrt{5})}$
i) $\frac{x^2-3x}{x(y-3)} \cdot \frac{xy^2-9x}{x-3}$ j) $\frac{(3x-y)^2+6xy}{9x^2+y^2}$

Problem 3 Write as one fraction in the simplest way possible.

a) $\frac{2}{x} + \frac{3}{x}$ b) $\frac{2}{x} + \frac{3}{y}$ c) $\frac{x-4}{x} - \frac{x}{x-4}$ d) $x+3 + \frac{2}{x-1}$
e) $\frac{1}{\sqrt{7}-1} - \frac{1}{\sqrt{7}+1}$ f) $\frac{1}{x-1} - \frac{1}{x+1}$ g) $\frac{x+3}{x^2+1} - \frac{1}{x-3}$ h) $\frac{x(x+3)}{(x+1)(x+2)} - 1$

Problem 4 If you replace y with x in the answer to problem 3b you should get the answer in problem 3a. Why? Is it true?

What do you get if x is replaced by $\sqrt{7}$ in the answer to problem 3f? Compare with problem 3e.

Problem 5 Compute without using digital assistance.

a) 2^3 b) 3^2 c) -3^2 d) $(-3)^2$ e) -2^3 f) $(-2)^3$

Problem 6 Compute/simplify without using digital assistance.

a) $\frac{4^2}{2^6}$ b) $\frac{2^5 \cdot 10^3}{5^3}$ c) $\frac{3^{10}}{3^9}$ d) $\frac{3^9}{3^{10}}$ e) $\frac{35^8}{5^7 \cdot 7^8}$

f) $\left(\frac{1}{2}\right)^3$ g) $\left(\frac{2}{5}\right)^3$ h) $(\sqrt{7})^2$ i) $(\sqrt{7})^3$ j) $\frac{7^2}{\sqrt{7} \cdot 7}$

k) $\frac{6^2}{\sqrt{2} \cdot \sqrt{3}}$ l) $\frac{(\sqrt{1,03})^{10}}{1.03^4}$ m) $\sqrt{3^2 + 4^2}$ n) $(\sqrt[3]{5})^3$ o) $(\sqrt[4]{7})^8$

p) $\sqrt[6]{27}$ q) $| -3.2 |$ r) $| 4.3 - 5.9 |$

Problem 7 Solve the equations.

a) $x^2 = 9$ b) $x^2 = -9$ c) $\sqrt{x} = 9$ d) $\sqrt{x} = -9$

e) $(x - 4)^2 = 9$ f) $(x + 7)^3 = 27$ g) $|x| = 25$ h) $|x| = -1$

i) $\frac{1}{|x|} = 0.25$ j) $x|x| = 9$ k) $|x - 2| = 25$ l) $|x - 3| = 3$

Problem 8 Solve the equations (a is an arbitrary number).

a) $x^2 = 3^2$ b) $x^2 = (-3)^2$ c) $x^2 = a^2$

d) $x^3 = 2^3$ e) $x^3 = (-2)^3$ f) $x^3 = a^3$

g) $x^2 = a^{-4}$ h) $x^3 = 1.03^{-12}$ i) $x^4 = a^{-4}$

Answers

Problem 1

- | | | | | | |
|-------------------|-------------------|--------------------|-------------------|-------------------|------------------|
| a) $\frac{3}{10}$ | b) $\frac{5}{4}$ | c) $\frac{32}{23}$ | d) $\frac{28}{3}$ | e) $\frac{1}{16}$ | f) $\frac{7}{2}$ |
| g) $\frac{8}{15}$ | h) $\frac{4}{35}$ | i) $\frac{1}{4}$ | j) $\frac{2}{9}$ | k) 1 | l) $\frac{4}{9}$ |

Problem 2

- | | | | | |
|-------------------|--------------------|--------------------|---------------------------|-----------------------|
| a) $\frac{x}{yz}$ | b) $\frac{5a}{3c}$ | c) $\frac{x}{x+2}$ | d) $\frac{x^2+1}{x(x+1)}$ | e) $\frac{1}{3}(x+y)$ |
| f) $x+y$ | g) $\frac{1}{2}$ | h) 1 | i) $x(y+3)$ | j) 1 |

Problem 3

- | | | | |
|------------------|---------------------------|-------------------------------|----------------------------|
| a) $\frac{5}{x}$ | b) $\frac{3x+2y}{xy}$ | c) $\frac{8(2-x)}{x(x-4)}$ | d) $\frac{x^2+2x-1}{x-1}$ |
| e) $\frac{1}{3}$ | f) $\frac{2}{(x-1)(x+1)}$ | g) $\frac{-10}{(x^2+1)(x-3)}$ | h) $\frac{-2}{(x+1)(x+2)}$ |

Problem 5

- | | | | | | |
|------|------|-------|------|-------|-------|
| a) 8 | b) 9 | c) -9 | d) 9 | e) -8 | f) -8 |
|------|------|-------|------|-------|-------|

Problem 6

- | | | | | | |
|----------------------|----------|----------------|------------------|----------------|------------------|
| a) $\frac{1}{4}$ | b) 2^8 | c) 3 | d) $\frac{1}{3}$ | e) 5 | f) $\frac{1}{8}$ |
| g) $\frac{2^3}{5^3}$ | h) 7 | i) $7\sqrt{7}$ | j) $\sqrt{7}$ | k) $6\sqrt{6}$ | l) 1,03 |
| m) 5 | n) 5 | o) 49 | p) $\sqrt{3}$ | q) 3,2 | r) 1,6 |

Problem 7

- | | | | |
|--------------------------|----------------|----------------------------|-------------------------|
| a) $x = 3, \quad x = -3$ | b) no solution | c) $x = 81$ | d) no solution |
| e) $x = 1, \quad x = 7$ | f) $x = -4$ | g) $x = 25, \quad x = -25$ | h) no solution |
| i) $x = 4, \quad x = -4$ | j) $x = 3$ | k) $x = -23, \quad x = 27$ | l) $x = 0, \quad x = 6$ |

Problem 8

- | | | |
|------------------------------------|--------------------------|--|
| a) $x = 3, \quad x = -3$ | b) $x = 3, \quad x = -3$ | c) $x = a , \quad x = - a $, which is the same as
$x = \pm a$ |
| d) $x = 2$ | e) $x = -2$ | f) $x = a$ |
| g) $x = a^{-2}, \quad x = -a^{-2}$ | h) $x = 0,888$ | i) $x = \frac{1}{a}, \quad x = -\frac{1}{a}$ |