

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 10

Sec. 4.1-6: Functions and graphs.

Linear and quadratic functions. Revenue and cost functions.

Here are recommended exercises from the textbook [SHSC].

Section 4.2 exercise 1-6, 13-15

Section 4.3 exercise 1-3

Section 4.4 exercise 1-10

Section 4.6 exercise 1-7

Problems for the exercise session Wednesday 28 Sept. from 12 in B2-065

Problem 1 Determine the expression of the linear function $f(x)$ in figure 1.

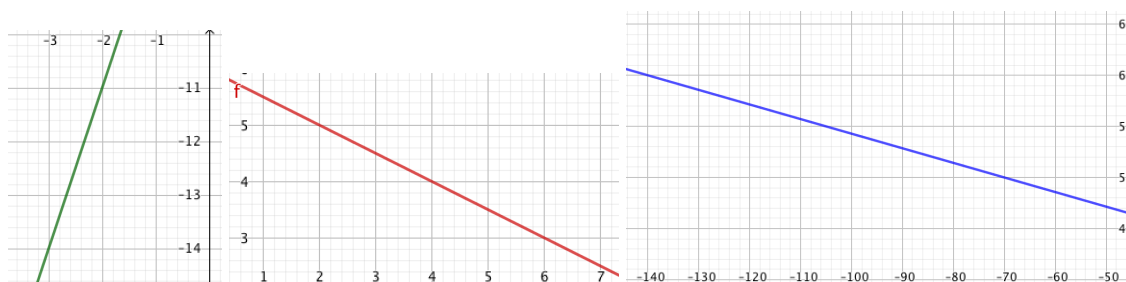


Figure 1: Linear functions (a-c)

Problem 2 Determine the intersection points of the graph and the x -axis and of the graph and the y -axis in Problem 1 a-c.

Problem 3 Determine the expression of the linear function $f(x)$ such that the graph passes through the points P and Q .

a) $P = (-2, 5)$ and
 $Q = (-4, 6)$

b) $P = (80, 90)$ and
 $Q = (50, 80)$

c) $P = (4, -3)$ and
 $Q = (-1, 7)$

Problem 4 Determine the expression of the linear function $f(x)$ such that the graph passes through the point P and has slope a .

a) $P = (-2, 5)$ and $a = \frac{2}{3}$

b) $P = (8, 90)$ and $a = \frac{1}{10}$

c) $P = (4, 30)$ and $a = -\frac{3}{10}$

Problem 5 Determine the expression of the second degree polynomial function $f(x)$ in a-f, see figure 2 and 3.

Problem 6 Determine the intersection points of the graph and the x -axis and of the graph and the y -axis in Problem 5 c-f.

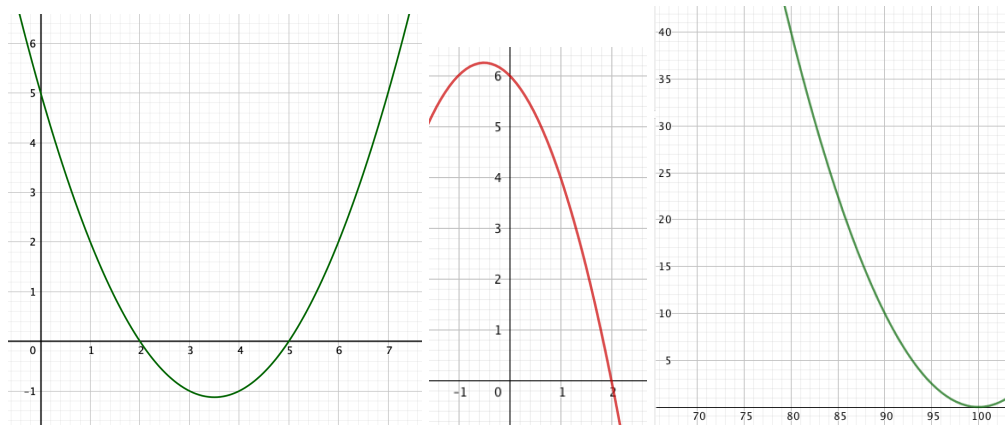


Figure 2: Parabolas (a-c)

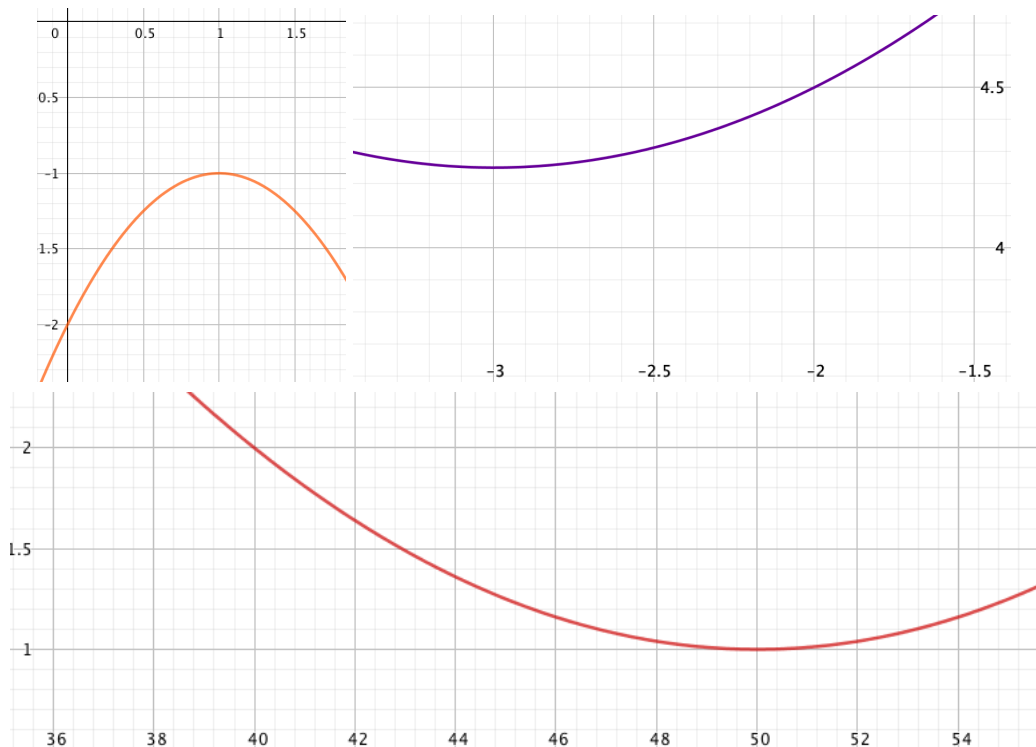


Figure 3: Parabolas (d-f)

Problem 7 Determine the expression of the second degree polynomial function $f(x)$ such that:

- a) The graph passes through the points $P = (0, 7)$, $Q = (1, 4)$ and $R = (2, 3)$.
- b) The graph passes through the points $P = (-5, 65)$, $Q = (3, 65)$ and $R = (7, 17)$.
- c) The graph passes through the point $P = (4, -6)$ and $Q = (\frac{13}{2}, -\frac{49}{4})$ is the point where the function attains its minimum.

Problem 8 Write $f(x)$ in the form $a(x - s)^2 + d$ (by ‘completing the square’) and use the expression to sketch the graph. In particular determine the symmetry axis of the parabola and the maximum or minimum value.

- a) $f(x) = x^2 - 10x + 30$
- b) $f(x) = 3x^2 + 36x + 110$
- c) $f(x) = -\frac{1}{7}x^2 + 2x - 6$

Problem 9 Suppose x is the number of units produced and sold. Determine the value of p (the unit price) which gives positive profit for $x > 300$ and negative profit for $x < 300$ if:

- a) The cost function is $C(x) = 2100 + 5x$ and the revenue function is $R(x) = px$.
- b) The cost function is $C(x) = 4500 - 5x + 0.01x^2$ and the revenue function is $R(x) = px$ (both with $0 \leq x \leq 1000$ as domain of definition).

Answers

Problem 1

- a) $f(x) = 3x - 5$
- b) $f(x) = -\frac{x}{2} + 6$
- c) $f(x) = -\frac{x}{7} + 40$

Problem 2

- a) $x = \frac{5}{3}$ and $y = -5$
- b) $x = 12$ and $y = 6$
- c) $x = 280$ and $y = 40$

Problem 3

- a) $f(x) = -\frac{1}{2}x + 4$
- b) $f(x) = \frac{1}{3}x + \frac{190}{3}$
- c) $f(x) = -2x + 5$

Problem 4

- a) $f(x) = \frac{2}{3}x + \frac{19}{3}$
- b) $f(x) = \frac{1}{10}x + \frac{446}{5}$
- c) $f(x) = -\frac{3}{10}x + \frac{156}{5}$

Problem 5

- a) $f(x) = \frac{1}{2}(x-2)(x-5) = \frac{1}{2}x^2 - \frac{7}{2}x + 5$
- b) $f(x) = -(x+3)(x-2) = -x^2 - x + 6$
- c) $f(x) = \frac{1}{10}(x-100)^2 = \frac{1}{10}x^2 - 20x + 1000$
- d) $f(x) = -(x-1)^2 - 1 = -x^2 + 2x - 2$
- e) $f(x) = \frac{1}{4}(x+3)^2 + \frac{17}{4} = \frac{1}{4}x^2 + \frac{3}{2}x + \frac{13}{2}$
- f) $f(x) = \frac{1}{100}(x-50)^2 + 1 = \frac{1}{100}x^2 - x + 26$

Problem 6

- c) $x = 100$ and $y = 1000$
- d) non and $y = -2$
- e) non and $y = \frac{13}{2}$
- f) non and $y = 26$

Problem 7

- a) $(x-2)^2 + 3 = x^2 - 4x + 7$
- b) $-(x+1)^2 + 81 = -x^2 - 2x + 80$
- c) $(x - \frac{13}{2})^2 - \frac{49}{4} = x^2 - 13x + 30$

Problem 8

- a) $f(x) = (x-5)^2 + 5$, the vertical line $x = 5$, $y = 5$ is the minimum value
- b) $f(x) = 3(x+6)^2 + 2$, the vertical line $x = -6$, $y = 2$ is the minimum value
- c) $f(x) = -\frac{1}{7}(x-7)^2 + 1$, the vertical line $x = 7$, $y = 1$ is the maximum value

For sketches of a-c see figure 4. A small table with relevant function values is expected.

Problem 9

- a) $p = 3600/300 = 12$
- b) $p = 13$

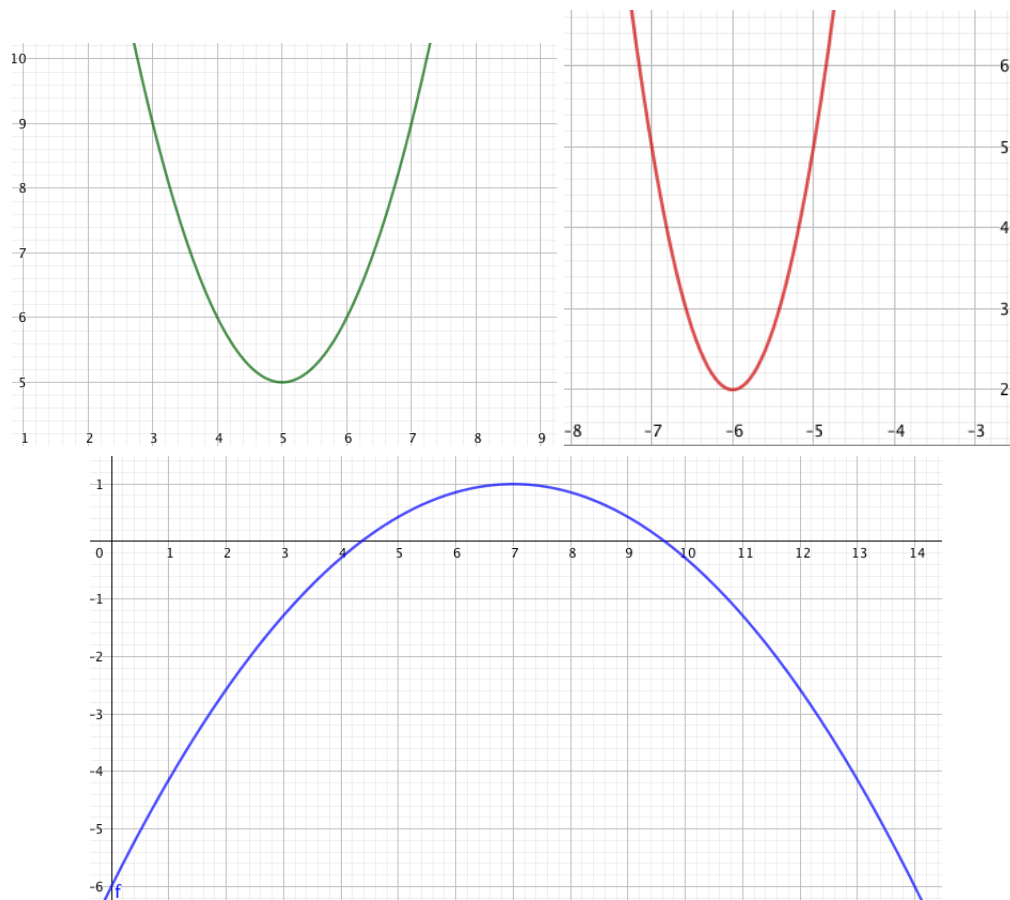


Figure 4: Parabolas in Problems 8 a-c