EBA1180 Mathematics for Data Science
autumn 2022
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

R. Lucas

Lecture 13
Sec. 4.7, 7.9: Rational functions and asymptotes.

Here are recommended exercises from the textbook [SHSC].

Section **4.7** exercise 4
Section **7.9** exercise 1-5
Section **5.2** exercise 2a, 3, 4
Section **5.3** exercise 1, 3-5, 7, 9, 10
Section **4.9** exercise 1, 2, 4, 6
Section **4.10** exercise 1, 2, 6, 8-10

Problems for the exercise session
Wednesday 12 Oct. from 12-17 in B2-065

Problem 1 Determine the expression $f(x) = c + \frac{a}{x-b}$ of the hyperbolas (a-d) in figure 1.

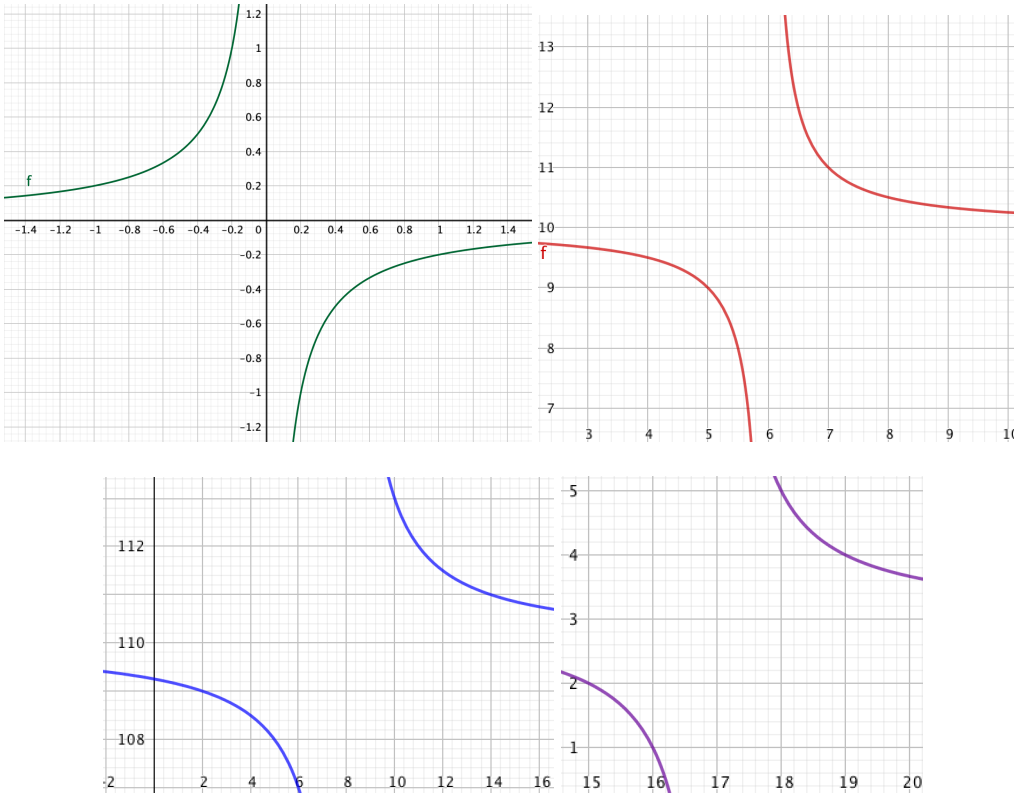

Figure 1: Hyperbolas a-d

Problem 2 Determine the asymptotes of the hyperbolas (a-d) in Problem 1.

Problem 3 Determine the asymptotes of the rational functions.

a) $f(x) = \frac{4x-10}{x-3}$ b) $f(x) = \frac{70-40x}{3-2x}$ c) $f(x) = \frac{12}{x^2+3}$

d) $f(x) = \frac{4x^2-28x+40}{x^2-4x+3}$ e) $f(x) = \frac{x^2+3x+5}{x-7}$ f) $f(x) = \frac{x^3-8}{x^2-10x+16}$

Answers

Problem 1

a) $f(x) = -\frac{1}{5x}$ b) $f(x) = 10 + \frac{1}{x-6}$ c) $f(x) = 110 + \frac{6}{x-8}$ d) $f(x) = 3 + \frac{2}{x-17}$

Problem 2

a) vertical asymptote: $x = 0$, horizontal asymptote: $y = 0$
b) vertical asymptote: $x = 6$, horizontal asymptote: $y = 10$
c) vertical asymptote: $x = 8$, horizontal asymptote: $y = 110$
d) vertical asymptote: $x = 17$, horizontal asymptote: $y = 3$

Problem 3

a) $f(x) = 4 + \frac{2}{x-3}$ so vertical asymptote: $x = 3$, horizontal asymptote: $y = 4$
b) $f(x) = 20 - \frac{10}{2x-3}$ so vertical asymptote: $x = \frac{3}{2}$, horizontal asymptote: $y = 20$
c) Since $x^2 + 3$ is positive for all x, $f(x)$ is defined for all x, so no vertical asymptote. Horizontal asymptote: $y = 0$
d) $f(x) = 4 - \frac{4(3x-7)}{(x-1)(x-3)}$ so vertical asymptotes: $x = 1$ and $x = 3$, horizontal asymptote: $y = 4$
e) $f(x) = x + 10 + \frac{75}{x-7}$ so vertical asymptote: $x = 7$, non-vertical asymptote: $y = x + 10$
f) $f(x) = x + 10 + \frac{84}{x-8}$ so vertical asymptote: $x = 8$, non-vertical asymptote: $y = x + 10$