EBA1180 Mathematics for Business Analytics autumn 2022

Exercises

I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.

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Lecture 15 – 16

Sec. 6.1-2, 6.6-8: Tangents, differentiation and rules for differentiation.

Here are recommended exercises from the textbook [SHSC].

Section **6.1** exercise 1, 2
Section **6.2** exercise 1, 6, 8
Section **6.6** exercise 1, 3
Section **6.7** exercise 1-4, 7
Section **6.8** exercise 1a, 10

Problems for the exercise session
Wednesday 26 Oct. at 12-17 in B2-065

Problem 1 Make a sketch of the graphs of two different functions $f(x)$ with the given data. Note: You are not supposed to find any algebraic expression!
a) $f(5) = 10$, $f'(5) = -1$
b) $f(3) = 5$, $f'(3) = 2$, $f(5) = 5$, $f'(5) = 0$
c) $f(10) = 100$, $f'(10) = 0.5$, $f(20) = 40$, $f'(20) = 2$, $f'(30) = 0$
d) $f(1) = 3$, $f'(3) = -0.2$, $f(5) = 4$, $f'(7) = \frac{2}{3}$

Problem 2 Suppose $f(x) = g(x) \cdot h(x)$. Use the product rule $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$ to find the derivative of $f(x)$ if:

a) $g(x) = 22x - 3$ and $h(x) = 3 - 7x$
b) $g(x) = x^{10} - 1$ and $h(x) = 3x^8 - 8x + 5$
c) $g(x) = x^{-3.5}$ and $h(x) = 3x^6 - 5x^5 + x^4$
d) $g(x) = \dfrac{1}{x^2}$ and $h(x) = x^4 - 4x + 230$
e) $g(x) = x^3 - \dfrac{1}{x^3}$ and $h(x) = 3\sqrt{x}$
f) $g(x) = 3x$ and $h(x) = 2e^x$
g) $g(x) = x$ and $h(x) = \ln(x)$
h) $g(x) = 5x\ln(x)$ and $h(x) = 6xe^x$

Problem 3 Suppose $f(x) = \dfrac{g(x)}{h(x)}$. Use the quotient rule $f'(x) = \dfrac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{[h(x)]^2}$ to find the derivative of $f(x)$ if:

a) $g(x) = 11x - 3$ and $h(x) = 3 - 7x$
b) $g(x) = x + 5$ and $h(x) = 9x - 1$
c) $g(x) = 3x^2 + 1$ and $h(x) = x - 10$
d) $g(x) = x^6$ and $h(x) = x^4 + 1$
e) $g(x) = x^{1,2}$ and $h(x) = 5x^2 - 1$
f) $g(x) = 5$ and $h(x) = x^2 - 4x + 10$
g) $g(x) = 5\ln(x)$ and $h(x) = x^2 + 3$
h) $g(x) = 2\ln(x)$ and $h(x) = 3e^x$
i) $g(x) = \ln(x) + 1$ and $h(x) = \ln(x) + 2$
j) $g(x) = e^x + 1$ and $h(x) = e^x + 2$

Problem 4 In figure 1 you see the graph of $f(x)$.

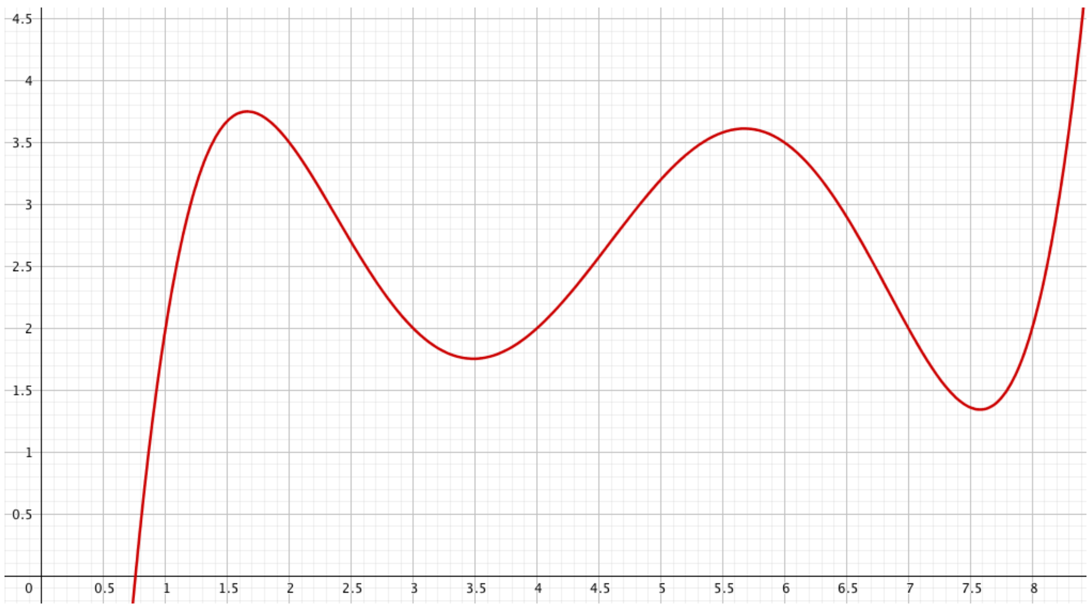

Figure 1: The graph of $f(x)$

Determine if the statement is true or false.

a) $f'(2) < f'(1)$

b) $f'(3) < f'(6.5)$

c) $f'(4.5) < f'(5.1)$

d) $f'(2.5) < f'(3)$

e) $f'(x)$ is positive for $6 < x < 7.5$

f) $f'(x)$ has no maximum points

g) $f'(x)$ has 4 zeros

h) $f'(x)$ is increasing in the interval $[3,4]$

i) $f'(x)$ is decreasing in the interval $[1,2]$

j) $f'(3) = 2$

k) $f'(x)$ has a minimum point in the interval $[2,3]$

Problem 5 Determine the expressions for $f(x)$, $u(x)$, $g(u)$, $u'(x)$ and $g'(u)$ which are not given in the table such that $f(x) = g(u(x))$. Use the chain rule $f'(x) = g'(u(x)) \cdot u'(x)$ to find $f'(x)$.

$f(x)$	$u(x)$	$g(u)$	$u'(x)$	$g'(u)$	$f'(x)$
$(3x+5)^2$	$3x+5$	u^2			
$2(x^2+3)^7+4$	x^2+3				
$7\sqrt{3x-1}$				$\dfrac{7}{2\sqrt{u}}$	
	x^2+10	$3e^u$			
$\ln(4x^2+5)$			$8x$		
$9(4x^3+1)^{3.5}$					
$3\left(\dfrac{4x-1}{9x+2}\right)^7$					
$50e^{-0.03x}$					
$\ln(1+e^{-x})$					
$\dfrac{2}{(2x+1)\sqrt{2x+1}}$					

Problem 6 Determine $f'(a)$.

a) $f(x) = g(x)h(x)$, $a = 10$, $g(10) = 20$, $g'(10) = 0.2$ and $h(10) = 60$, $h'(10) = 0.5$.

b) $f(x) = \frac{g(x)}{h(x)}$, $a = 7$, $g(7) = 20$, $g'(7) = 0.2$ and $h(7) = 10$, $h'(7) = 0.05$.

c) $f(x) = g(u(x))$, $a = 3$, $g(3) = 12$, $g'(3) = -0.6$, $g(10) = 20$, $g'(10) = 1.07$, $u(10) = 1$, $u'(10) = 0$, $u(3) = 10$, $u'(3) = 2$.

Problem 7 Determine which is the larger number:

a) 3^{5000} or 4^{4000} b) 1.02^{4321} or 1.025^{3478} c) 1.12^{1000} or 1.01^{12000}

Problem 8 (Multiple choice spring 2016, problem 10)
We have the function $f(x) = x^2 e^{2-x} - e\ln(\sqrt{e})$. The slope a for tangent of f in $x = 2$ is:

(A) $a = 2$
(B) $a = \frac{3}{2}$
(C) $a = 0$
(D) $a < 0$
(E) I choose not to solve this problem.

Answers

Problem 1
Compare with other students, ask the learning assistants!

Problem 2

a) $87 - 308x$

b) $54x^{17} - 88x^{10} + 50x^9 - 24x^7 + 8$

c) $7.5 \cdot x^{1.5} - 7.5 \cdot x^{0.5} + 0.5 \cdot x^{-0.5}$

d) $2x + 4x^{-2} - 460x^{-3}$

e) $10.5 \cdot x^{2.5} + 7.5 \cdot x^{-3.5}$

f) $6(x+1)e^x$

g) $\ln(x) + 1$

h) $30x[x\ln(x) + 2\ln(x) + 1]e^x$

Problem 3

a) $\dfrac{12}{(3-7x)^2}$ b) $-\dfrac{46}{(9x-1)^2}$ c) $\dfrac{3x^2-60x-1}{(x-10)^2}$ d) $\dfrac{2x^5(x^4+3)}{(x^4+1)^2}$

e) $-\dfrac{x^{0.2}(4x^2+1.2)}{(5x^2-1)^2}$ f) $-\dfrac{10(x-2)}{(x^2-4x+10)^2}$ g) $\dfrac{5[x^2+3-2x^2\ln(x)]}{x(x^2+3)^2}$ h) $\dfrac{2[1-x\ln(x)]}{3xe^x}$

i) $\dfrac{1}{x[\ln(x)+2]^2}$ j) $\dfrac{e^x}{(e^x+2)^2}$

Problem 4

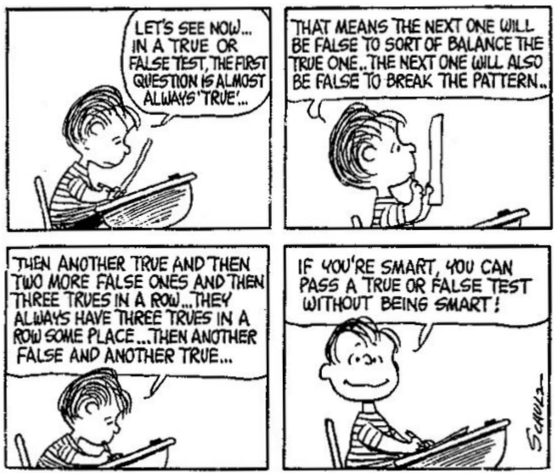

Figure 2: True or false

Problem 5

$f(x)$	$u(x)$	$g(u)$	$u'(x)$	$g'(u)$	$f'(x)$
$(3x+5)^2$	$3x+5$	u^2	3	$2u$	$18x+30$
$2(x^2+3)^7+4$	x^2+3	$2u^7+4$	$2x$	$14u^6$	$28x(x^2+3)^6$
$7\sqrt{3x-1}$	$3x-1$	$7\sqrt{u}$	3	$\frac{7}{2\sqrt{u}}$	$\frac{10.5}{\sqrt{3x-1}}$
$3e^{x^2+10}$	x^2+10	$3e^u$	$2x$	$3e^u$	$6xe^{x^2+10}$
$\ln(4x^2+5)$	$4x^2+5$	$\ln(u)$	$8x$	u^{-1}	$\frac{8x}{4x^2+5}$
$9(4x^3+1)^{3.5}$	$4x^3+1$	$9u^{3.5}$	$12x^2$	$31.5u^{2.5}$	$378x^2(4x^3+1)^{2.5}$
$3\left(\frac{4x-1}{9x+2}\right)^7$	$\frac{4x-1}{9x+2}$	$3u^7$	$\frac{17}{(9x+2)^2}$	$21u^6$	$357\cdot\frac{(4x-1)^6}{(9x+2)^8}$
$50e^{-0.03x}$	$-0.03x$	$50e^u$	-0.03	$50e^u$	$-1.5e^{-0.03x}$
$\ln(1+e^{-x})$	$1+e^{-x}$	$\ln u$	$-e^{-x}$	u^{-1}	$-\frac{e^{-x}}{1+e^{-x}}$
$\frac{2}{(2x+1)\sqrt{2x+1}}$	$2x+1$	$2u^{-1.5}$	2	$-3u^{-2.5}$	$-6(2x+1)^{-2.5}$

Problem 6

a) $12+10=22$ b) $\frac{2-1}{10^2}=0.01$ c) $f'(3)=g'(u(3))\cdot u'(3)=1.07\cdot 2=2.14$

Problem 7

a) $3^{5000}=(3^5)^{1000}=243^{1000}$ while $4^{4000}=(4^4)^{1000}=256^{1000}$

b) $\ln(1.02^{4321})=4321\cdot\ln(1.02)=85.57$ and $\ln(1.025^{3478})=3478\cdot\ln(1.025)=85.88$. Because $\ln(x)$ is a strictly increasing function it follows that $1.02^{4321}<1.025^{3478}$.

c) 1000 years with 12% interest and annual compounding gives a smaller total growth factor than 1000 years with 12% interest and monthly compounding.

Problem 8

C