EBA1180 Mathematics for Data Science autumn 2022
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing. I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.

R. Lucas

Lecture 23 – 24
Sec. 7.7, 7.4-5: Elasticity. Linearisation. Taylor polynomials.

Section **7.7** exercise 1-3
Section **7.4** exercise 1-3
Section **7.5** exercise 1-3

Multiple choice exam 2018a, problem 10, 13
Multiple choice exam 2019s, problem 11
Multiple choice exam 2019a, problem 11

Problems for the exercise session Wednesday 23 Nov. from 12 – 17 in B2-065

Problem 1 Let p be the price of a commodity and $D(p)$ the demand for the commodity with price p (so $D(p)$ is the number of sold units). Determine the relative change of price, the relative change of demand and the (average) elasticity of the demand with respect to price. Determine if the demand is elastic, inelastic, or unit elastic.
a) $D(30) = 40$ and $D(30.5) = 39$
b) $D(20) = 101$ and $D(21) = 100.95$
c) $D(10) = 24.648$ and $D(10.01) = 24.623$

Problem 2 Let p be the price of a commodity and $D(p)$ the demand for the commodity with price p (so $D(p)$ is the number of sold units). Determine the (momentary) elasticity $\varepsilon(p) = \mathrm{El}_p(D(p))$ of the demand with respect to price. Determine the price p such that the demand is elastic, inelastic, and unit elastic.
a) $D(p) = 100 - 2p$ with $0 < p < 50$
b) $D(p) = 100 + \frac{20}{p}$ with $p \geqslant 1$
c) $D(p) = 67e^{-0.1p}$ with $p > 0$
d) $D(p) = 100 + \frac{900}{p^2}$ with $p \geqslant 1$
e) $D(p) = 53e^{-0.02p^2}$ with $p > 0$

Problem 3
a) Determine the Taylor polynomials $P_1(x), \ldots, P_4(x)$ of degree $1 - 4$ of the function $f(x) = e^x$ about 0.
b) Compute $P_1(1), \ldots, P_4(1)$ and compute how good approximations these values give to $f(1) = e$.

Problem 4
a) Determine the Taylor polynomials $P_1(x), \ldots, P_4(x)$ of degree $1 - 4$ of the function $f(x) = \ln(x)$ about 1.
b) Compute $P_1(2), \ldots, P_4(2)$ and compute how good approximations these values give to $f(2) = \ln(2)$.

Problem 5 We have a function $f(x)$ with $f(50) = 100$, $f'(50) = 1$ and $f''(50) = -0.4$.
a) Determine the Taylor polynomial $P_2(x)$ of $f(x)$ about 50.
b) Use $P_2(x)$ to give an approximate value for $f(52)$.

Problem 6 Let $P_1(x), P_2(x), P_3(x)$ be the three first Taylor polynomials in problem 3. Compute the limit values.

$$\text{a)}\ \lim_{x\to 0}\frac{e^x - P_1(x)}{x^2} \qquad \text{b)}\ \lim_{x\to 0}\frac{e^x - P_2(x)}{x^3} \qquad \text{c)}\ \lim_{x\to 0}\frac{e^x - P_3(x)}{x^4}$$

Problem 7 Let $P_1(x), P_2(x), P_3(x)$ be the three first Taylor polynomials in problem 4. Compute the limit values.

$$\text{a)}\ \lim_{x\to 1}\frac{\ln(x) - P_1(x)}{(x-1)^2} \qquad \text{b)}\ \lim_{x\to 1}\frac{\ln(x) - P_2(x)}{(x-1)^3} \qquad \text{c)}\ \lim_{x\to 1}\frac{\ln(x) - P_3(x)}{(x-1)^4}$$

Problem 8 (Multiple choice exam 2017s, problem 12)
We consider the price elasticity $\varepsilon = \varepsilon(p)$ of the demand function $D(p) = 120 - 8p$ of a commodity. Then:
(A) $\varepsilon > -1$ for $p = 7.5$
(B) $\varepsilon > -1$ for $p < 7.5$
(C) $\varepsilon > -1$ for $p > 7.5$
(D) $\varepsilon > -1$ for all values of p
(E) I choose not to answer this problem.

Problem 9 (Multiple choice exam 2016a, problem 12)
Demand for a commodity is given as $D(p) = 110 - 5p$. Then the elasticity $\varepsilon(p) = -1$ for:
(A) $p = 7$
(B) $p = 11$
(C) $p = \frac{16}{5}$
(D) $p = 22$
(E) I choose not to answer this problem.

Problem 10 (Home exam 2021a, problem 3)
Let p be the price of a commodity with demand function $D(p) = 100e^{-0.2p}$. Determine which prices p which make the demand elasitic, inelastic and unit elastic.

Problem 11 (Home exam 2021a, problem 10)
The function $f(x)$ has $f(30) = 700$, $f'(30) = 5$ and $f''(30) = -1$. Compute an approximate value for $f(31)$.

Answers

Problem 1

a) the relative change of price is $\frac{0.5}{30}$, the relative change of demand is $\frac{-1}{40}$ and the elasticity is -1.5, i.e. elastic demand
b) the relative change of price is $\frac{1}{20}$, the relative change of demand is $\frac{-0.05}{101}$ and the elasticity is -0.0099, i.e. inelastic demand
c) the relative change of price is 0.001, the relative change of demand is -0.001014 and the elasticity is -1.014, i.e. elastic demand

Problem 2

a) $\varepsilon(p) = \frac{-2p}{100-2p}$. The demand function is unit elastic for $p = 25$, inelastic for $0 < p < 25$ and elastic for $25 < p < 50$.
b) $\varepsilon(p) = -\frac{1}{5p+1}$. The demand function is inelastic for all $p \geqslant 1$.
c) $\varepsilon(p) = -0.1p$. The demand function is unit elastic for $p = 10$, inelastic for $0 < p < 10$ and elastic for $p > 10$.
d) $\varepsilon(p) = -\frac{18}{p^2+9}$. The demand function is unit elastic for $p = 3$, elastic for $1 \leqslant p < 3$ and inelastic for $p > 3$.
e) $\varepsilon(p) = -0.04p^2$. The demand function is unit elastic for $p = 5$, inelastic for $0 < p < 5$ and elastic for $p > 5$.

Problem 3

a) $P_1(x) = 1 + x$, $P_2(x) = 1 + x + \frac{x^2}{2}$, $P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, $P_4(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$
b) $P_1(1) = 2$, $P_2(1) = 2.5$, $P_3(1) = \frac{8}{3} \approx 2.67$, $P_4(x) = \frac{65}{24} \approx 2.71$. The distance from $f(1) = e$ equals (approximately):
$|f(1) - P_1(1)| = |e - 2| = 0.72$
$|f(1) - P_2(1)| = |e - 2.5| = 0.22$
$|f(1) - P_3(1)| = |e - \frac{8}{3}| = 0.052$
$|f(1) - P_4(1)| = |e - \frac{65}{24}| = 0.0099$

Problem 4

a) $P_1(x) = (x-1)$, $P_2(x) = (x-1) - \frac{(x-1)^2}{2}$, $P_3(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$,
$P_4(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$
b) $P_1(2) = 1$, $P_2(2) = \frac{1}{2}$, $P_3(1) = \frac{5}{6} \approx 0.83$, $P_4(x) = \frac{7}{12} \approx 0.58$. The distance from $f(2) = \ln(2)$ equals (approximately):
$|f(2) - P_1(2)| = |\ln(2) - 1| = 0.31$
$|f(2) - P_2(2)| = |\ln(2) - \frac{1}{2}| = 0.19$
$|f(2) - P_3(2)| = |\ln(2) - \frac{5}{6}| = 0.14$
$|f(2) - P_4(2)| = |\ln(2) - \frac{7}{12}| = 0.11$

Problem 5

a) $P_2(x) = 100 + (x-50) - 0.2(x-50)^2$
b) $f(52) \approx P_2(52) = 101.2$

Problem 6

a) This is a $\frac{0}{0}$-expression. Hence we can use l'Hôpital's rule. Differentiate numerator and denominator. Get another $\frac{0}{0}$-expression and use l'Hôpital's rule again:

$$\lim_{x\to 0} \frac{e^x - (1+x)}{x^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x - 1}{2x} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2} \quad \left(= \frac{f''(0)}{2}\right)$$

b)

$$\lim_{x\to 0} \frac{e^x - (1 + x + \frac{x^2}{2})}{x^3} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x - (1+x)}{3x^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x - 1}{6x} \overset{\text{l'Hôp}}{=} \lim_{x\to 0} \frac{e^x}{6} = \frac{1}{6} \quad \left(= \frac{f'''(0)}{3!}\right)$$

c) $\frac{1}{24}$ $\left(= \frac{f^{(4)}(0)}{4!}\right)$

Problem 7

a) This is a $\frac{0}{0}$-expression. Hence we can use l'Hôpital's rule. Differentiate numerator and denominator. Get another $\frac{0}{0}$-expression and use l'Hôpital's rule again:

$$\lim_{x\to 1} \frac{\ln(x)-(x-1)}{(x-1)^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{\frac{1}{x}-1}{2(x-1)} \overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{-\frac{1}{x^2}}{2} = -\frac{1}{2} \quad \left(= \frac{f''(1)}{2}\right)$$

b)

$$\lim_{x\to 1} \frac{\ln(x)-[(x-1)-\frac{(x-1)^2}{2}]}{(x-1)^3} \overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{\frac{1}{x}-[1-(x-1)]}{3(x-1)^2} \overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{-\frac{1}{x^2}+1}{6(x-1)} \overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{\frac{2}{x^3}}{6} = \frac{1}{3} \quad \left(= \frac{f'''(1)}{3!}\right)$$

c) $-\frac{1}{4}$ $\left(= \frac{f^{(4)}(1)}{4!}\right)$

Problem 8 (Multiple choice exam 2017s, problem 12)
B

Problem 9 (Multiple choice exam 2016a, problem 12)
B

Problem 10 (Home exam 2021a, problem 3)

$$\varepsilon(p) = -0{,}2p$$

The demand is *elastic* for $p > 5$.
The demand is *inelastic* for $0 < p < 5$.
The demand is *unit elastic* for $p = 5$.

Problem 11 (Home exam 2021a, problem 10)
The second degree Taylor polynomial for $f(x)$ about 30 is $P_2(x) = 700 + 5(x-30) - 0{,}5(x-30)^2$.
Then $f(31) \approx P_2(31) = 704{,}5$.