Exercise session problems

Problem 1.
Compute the definite integrals, and make figures to illustrate these as areas:

a) $\int_0^4 3 \, \mathrm{d}x$ b) $\int_0^8 (10+3x) \, \mathrm{d}x$ c) $\int_{-2}^2 |x| \, \mathrm{d}x$ d) $\int_{-1}^3 x - |x| \, \mathrm{d}x$

Problem 2.
Calculate the definite integrals:

a) $\int_0^1 x \, \mathrm{d}x$ b) $\int_0^1 x^2 \, \mathrm{d}x$ c) $\int_0^1 x^3 \, \mathrm{d}x$ d) $\int_0^1 e^x \, \mathrm{d}x$ e) $\int_0^1 (e^x + e^{-x}) \, \mathrm{d}x$

f) $\int_{-1}^1 x \, \mathrm{d}x$ g) $\int_{-1}^1 x^2 \, \mathrm{d}x$ h) $\int_{-1}^1 x^3 \, \mathrm{d}x$ i) $\int_{-1}^1 e^x \, \mathrm{d}x$ j) $\int_{-1}^1 (e^x + e^{-x}) \, \mathrm{d}x$

Problem 3.
Calculate the definite integrals:

a) $\int_0^1 xe^x \, \mathrm{d}x$ b) $\int_0^1 x\ln(x^2+1) \, \mathrm{d}x$ c) $\int_0^1 \frac{1}{x^2+5x+6} \, \mathrm{d}x$ d) $\int_0^1 \frac{1}{x^2+4x+4} \, \mathrm{d}x$

e) $\int_{-1}^1 xe^x \, \mathrm{d}x$ f) $\int_{-1}^1 x\ln(x^2+1) \, \mathrm{d}x$ g) $\int_{-1}^1 \frac{1}{x^2+5x+6} \, \mathrm{d}x$ h) $\int_{-1}^1 \frac{1}{x^2+4x+4} \, \mathrm{d}x$

Problem 4.
Find the area of R, and make a figure to illustrate R:

a) R is the area bounded by the graph of $y = \ln(2+x)$, the line $y = 2$, and the y-axis.
b) R is the area bounded by the graphs of $y = x$ and $y = x^2$.

Problem 5.
A company is renting out property. The company has an income stream from their tenants, currently this is 300 million NOK per year. We assume that the income stream will increase in the coming years, and choose the continuous function

$$f(t) = 300 \cdot e^{t/7}$$

as a model for the cash flow (in million NOK per year) after t years. Compute the total income during the next 10 years. How much of this income comes during the first two years?

Problem 6.
A company is renting out property. The company has an income stream from their tenants, currently this is 300 million NOK per year. We assume that the income stream will increase in the coming years, and choose the continuous function

$$f(t) = 300 \cdot e^{t/7}$$

as a model for the cash flow (in million NOK per year) after t years.

Compute the present value of the cash flow during the next 10 years. Use continuous compounding of interest, with a discount rate $r = 10\%$. What fraction of this present value comes from the rental payments during the first two years?

Problem 7.
The (inverse) demand function $p = f(q)$ and the (inverse) supply function $p = g(q)$ are given by

$$f(q) = 200 - 2q \quad \text{og} \quad g(q) = q + 20$$

Find the equilibrium price. Calculate the consumer surplus and the producer surplus, and illustrate these in a figure.

Problem 8.
The (inverse) demand function $p = f(q)$ and the (inverse) supply function $p = g(q)$ are given by

$$f(q) = \frac{6000}{q+50} \quad \text{og} \quad g(q) = q + 10$$

Find the equilibrium price. Calculate the consumer surplus and the producer surplus, and illustrate these in a figure.

Problem 9.
Write down a sum (based on at least $n = 10$ sub-intervals) which approximates the definite integral $\int_0^1 (1-x^2)\,\mathrm{d}x$, and illustrate the definite integrals and the approximation as areas in a figure.

Optional: Exercises from Norwegian textbook

Læreboken [E]:	Eriksen, *Matematikk for økonomi og finans*
Oppgaveboken [O]:	Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Oppgaver:	[E] 5.6.1 - 5.6.5, 5.7.1 - 5.7.2
Fullstendig løsning:	Se [O] Kap 5.6 - 5.7
Eksamensoppgaver	Se Oppgaveark 29

Answers to exercise session problems

Problem 1.
a) 12 b) 176 c) 4 d) -1

Problem 2.

a) $1/2$ b) $1/3$ c) $1/4$ d) $e-1$ e) $e-1/e$

f) 0 g) $2/3$ h) 0 i) $e-1/e$ j) $2(e-1/e)$

Problem 3.

a) 1 b) $\ln(2)-1/2$ c) $2\ln(3)-3\ln(2)$ d) $1/6$

e) $2/e$ f) 0 g) $\ln(3)-\ln((2)$ h) $2/3$

Problem 4.

a) $e^2-6+\ln(4)$ b) $1/6$

Problem 5.

The total income is $2100(e^{10/7}-1)\approx 6\,663$ million NOK. Out of this, $2100(e^{2/7}-1)\approx 694$ million NOK come from rental income during the first two years.

Problem 6.

The present value is $7000(e^{3/7}-1)\approx 3\,745$ million NOK. From this, $7000(e^{3/35}-1)\approx 626$ million NOK come from the rental payments during the first two years.

Problem 7.

The equilibrium price is $p^*=80$, the consumer surplus is $3\,600$ and the producer surplus is $1\,800$.

Problem 8.

The equilibrium price is $p^*=60$, the consumer surplus is $6000\ln(2)-3000\approx 1159$ and the producer surplus is 1250.

Problem 9.

If we partition the interval [0,1] into $n=10$ equal sub-intervals, the partition points become $x_i=i/10$ for $i=0,1,2,\ldots,10$. Hence, $x_0=0$, $x_1=1/10$, $x_2=2/10$ and so on. The definite integral is the area under $f(x)=1-x^2$ on the interval [0,1]. We can approximate this as the area of ten rectangles, given by the sum

$$\begin{aligned}\sum_{i=0}^{9} f(x_i)\cdot\Delta x_i &= \sum_{i=0}^{9}\left(1-(i/10)^2\right)\cdot\frac{1}{10} = (1+(1-1/100)+(1-4/100)+\cdots+(1-81/100))\cdot\frac{1}{10}\\ &= \frac{1}{10}\cdot\left(10-\frac{0+1+4+\cdots+81}{100}\right)=0.715\end{aligned}$$

The sum is shown in the figure below. The definite integral is the area under the blue curve, i.e. slightly less than 0.715. The choice of $n=10$ is not important, but the approximation improves as n increases.