EBA2910 Mathematics for Data Science
autumn 2022
Exercises

... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing.

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Lecture 6 and 7
Sec. 3.1-3, 4.7:
Linear and quadratic equations. Equations with parametres. Polynomial equations.

Here are recommended exercises from the textbook [SHSC].

Section **3.1** exercise 1-5
Section **3.2** exercise 2a, b, e, 4a, 5a, c
Section **3.3** exercise 1, 2, 6

Repetition (I have translated them, see It's Learning under «Resources»)
Multiple choice exam 2017 autumn, problem 1-4
Multiple choice exam 2018 spring, problem 1, 2 and 4

Problems for the exercise session
Wednesday 15 Sept. at 12-15 in D1-065 (or on Zoom)

Problem 1 Solve the linear equations for x.

a) $3x-5=0$
b) $ax+b=0$
c) $cx+50-c=0$ $(c\neq 0)$
d) $-4x+9=3-0{,}25x$
e) $ax+b=cx+d$
f) $\frac{x}{4}+11-k=3-\frac{x}{3}$
g) $22-4x+9-2(x+7)=14-5(x+2)+x-23$
h) $k-bx-d(3-5x)+11=c$
i) $\frac{4}{x-2}=\frac{3}{2x+1}$
j) $\frac{5-x}{x-a}=7$
k) $\frac{6}{3x+6}=\frac{14}{2x+4}$
l) $5-7x=4x+29-11x$
m) $3(x+1)=3x+3$

Problem 2 Solve the quadratic equations.

a) $x^2=0$
b) $x^2=1$
c) $x^2=-1$
d) $(x-3)^2=0$
e) $(x+3)^2=1$
f) $(x+3)^2=-1$
g) $x^2-6x+9=0$
h) $x^2-6x+9=1$
i) $x^2-6x+9=-1$
j) $(3x-1)^2=0$
k) $(3x-1)^2=9$
l) $(3x-1)^2=-9$
m) $9x^2-6x+1=0$
n) $9x^2-6x+1=9$
o) $9x^2-6x+1=-9$
p) $(x-2)(x+3)=0$
q) $(3x+5)(2x-1)=0$
r) $(x-2)(x+3)=-6$
s) $x^2-5x-2=0$
t) $2x^2+4x=70$
u) $x(x-8)=90-x^2$

Problem 3 Determine the quadratic equation $x^2 + bx + c = 0$ with the given solutions.

a) $x = 1$, $x = -1$ b) $x = 3$, $x = -2$ c) $x = -3$, $x = -8$

d) $x = 17$, $x = -3$ e) $x = 3 \pm \sqrt{5}$ f) $x = -11$

Problem 4 Solve the quadratic equations by *completing the square*.

a) $x^2 - 8x = -12$ b) $x^2 + 10x = -9$ c) $x^2 - 5x - 14 = 0$

d) $x^2 - 3x = 4$ e) $x^2 - 24x = 25$ f) $x^2 + 5x = 6$

Problem 5

a) Determine k such that the equation $x^2 - 10x = k$ has exactly one solution.
b) Determine k such that the equation $x^2 - kx = -9$ has exactly one solution.
c) Determine k such that the equation $\frac{1}{k}x^2 - 14x = 12$ has exactly one solution.

Problem 6 Write the quadratic expression as a product of two linear (grade 1) polynomials.

a) $x^2 - 8x + 12$ b) $x^2 + 10x + 9$ c) $x^2 - 18x + 81$

d) $2x^2 - 4x - 70$ e) $3x^2 + 24x - 99$ f) $5x^2 + 60x + 180$

Problem 7

a) Use one parameter to write an expression for all polynomials on the form $x^2 + bx + c$ which have two zeros of distance 1 from each other.
b) Use one parameter to write an expression for all third degree polynomials on the form $x^3 + ax^2 + bx + c$ which have three zeros with the middle one of distance 5 to each of the other two.
c) Use two parameters to write up all quadratic polynomials with exactly one zero.

Problem 8 Solve the equations for x.

a) $(2x - \sqrt{3})(x^2 - 20x + 99) = 0$
b) $(x^2 - 5)(x^2 - 6x + 4) = 0$
c) $(2x^2 - 28x + 98)(x^2 - 6x + 10)(x^2 - 2x - 6) = 0$
d) $(x + 1)(x + 2)(x + 3)(x + a)(x^2 + b) = 0$

Problem 9 Solve the equations.

a) $x^4 - 11x^2 + 18 = 0$
b) $x^6 + 13x^3 + 40 = 0$
c) $x^{100} - x^{50} - 12 = 0$

Problem 10 Solve the equations.

a) $1 + x + x^2 + x^3 + \cdots + x^{98} + x^{99} = 0$
b) $1 + x + x^2 + x^3 + \cdots + x^{49} + x^{50} = 0$
c) $\frac{x}{1.02} + \frac{x^2}{1.02^2} + \frac{x^3}{1.02^3} + \frac{x^{29}}{1.02^{29}} + \frac{x^{30}}{1.02^{30}} = 0$

Answers

Problem 1

a) $x = \frac{5}{3}$ b) $x = -\frac{b}{a}$ c) $x = \frac{c-50}{c} = 1 - \frac{50}{c}$ d) $x = \frac{8}{5}$

e) $x = \frac{d-b}{a-c}$ if $a \neq c$. For $a = c$ and $b \neq d$ there is no solution,
for $a = c$ and $b = d$ all numbers are solutions.

f) $x = \frac{12k-96}{7}$ g) $x = 18$

h) $x = \frac{c+3d-k-11}{5d-b}$ if $b \neq 5d$. If $b = 5d$ and $c + 3d \neq k + 11$ there are no solutions,
if $b = 5d$ and $c + 3d = k + 11$ all numbers are solutions.

i) $x = -2$ j) $x = \frac{7a+5}{8}$ k) no solutions l) no solutions

m) all numbers are solutions

Problem 2

a) $x = 0$ b) $x = 1, x = -1$ c) no solutions d) $x = 3$

e) either $x + 3 = 1$, i.e. $x = -2$,
or $x + 3 = -1$, i.e. $x = -4$ f) no solutions g) $x = 3$

h) $x = 2$, $x = 4$ i) no solutions j) $x = \frac{1}{3}$ k) $x = \frac{1}{3} \pm 1$, i.e.
$x = \frac{4}{3}$, $x = -\frac{2}{3}$

l) no solutions m) $x = \frac{1}{3}$ n) $x = \frac{4}{3}$, $x = -\frac{2}{3}$ o) no solutions

p) $x = 2$, $x = -3$ q) $x = -\frac{5}{3}$, $x = \frac{1}{2}$ r) $x = 0$, $x = -1$ s) $x = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$

t) $x = -7$, $x = 5$ u) $x = -5$, $x = 9$

Problem 3

a) $(x-1)(x-(-1)) = x^2 - 1$ b) $x^2 - x - 6$ c) $x^2 + 11x + 24$

d) $x^2 - 14x - 51$ e) $x^2 - 6x + 4$ f) $x^2 + 22x + 121$

Problem 4

a) $(x-4)^2 = -12 + 16$ gives
$x = 2$, $x = 6$ b) $(x+5)^2 = -9 + 25$ gives
$x = -9$, $x = -1$ c) $(x - \frac{5}{2})^2 = 14 + \frac{25}{4}$ gives
$x = -2$, $x = 7$

d) $(x - \frac{3}{2})^2 = 4 + \frac{9}{4}$ gives
$x = -1$, $x = 4$ e) $(x-12)^2 = 25 + 12^2$ gives
$x = -1$, $x = 25$ f) $(x + \frac{5}{2})^2 = 6 + \frac{25}{4}$ gives
$x = -6$, $x = 1$

Problem 5

a) $k = -25$ b) $k = \pm 6$ c) $k = -\frac{12}{49}$

Problem 6

a) $(x-2)(x-6)$ b) $(x+9)(x+1)$ c) $(x-9)^2$

d) $2(x+5)(x-7)$ e) $3(x+11)(x-3)$ f) $5(x+6)^2$

Problem 7

a) If r is the smallest zero, the polynomial is $x^2-(2r+1)x+r(r+1)$.
b) If r is the second smallest zero, the polynomial is $(x-r)^3-25(x-r)=x^3-3rx^2+(3r^2-25)x-(r-5)r(r+5)$.
c) $a(x-r)^2=ax^2-2arx+ar^2$ $(a\neq 0)$.

Problem 8

a) $x=\frac{\sqrt{3}}{2}$, $x=9$, $x=11$
b) $x=\pm\sqrt{5}$, $x=3\pm\sqrt{5}$
c) $x=7$, $x=1\pm\sqrt{7}$
d) $x=-1$, $x=-2$, $x=-3$, $x=-a$, $x=\pm\sqrt{-b}$ for $b\leqslant 0$

Problem 9

a) $x=\pm\sqrt{2}$, $x=\pm 3$ (Hint: Substitute $u=x^2$)
b) $x=-2$, $x=-\sqrt[3]{5}$
c) $x=\pm\sqrt[25]{2}$

Problem 10

a) $x=-1$
b) no solutions
c) $x=0$, $x=-1.02$