EBA 1180, lecture 1, 23 Aug. 2023, Runar Ile

Plan
1. Intro. to the Course
2. Algebraic expressions
3. Roots
4. Powers
5. Order of operations
6. Absolute value

1. Intro. to the Course

Autumn
- Financial math.
- Functions & graphs
- Differentiation and optimization

Spring
- Integration
- Systems of linear equations
- Functions in two variables $z = f(x,y)$

2. Algebraic expressions

Variables: x, y, z, x_1, x_2, x_3

$a, b, c, \dots, m, n$

Multiply with a number:

$3 \cdot x \overset{\text{short form}}{=} 3x$

$3 \cdot 2 \neq 32$

$\sqrt{3} \cdot x = \sqrt{3}x$

$(-1) \cdot x = -x$

$1 \cdot x = x$

$0 \cdot x = 0$

Addition

$x + x = 2x$

$x + y$ no simplification

$x + y + x = 2x + y$

Multiplication $x \cdot y = xy$

$x \cdot x = x^2$

$xy \cdot x^2 = x \cdot y \cdot x \cdot x = x^3 y$

Division $\underbrace{\frac{x+4y}{z}}_{\text{rat. ex.}}$, $\underbrace{\frac{2xy+\sqrt{3}}{3x+y^2}}_{\text{rational expression}}$ ← polynomial ← polynomial

Rational expression : fractions of polynomials

Other expressions : $\sqrt{x^2+1}$, $\frac{3\sqrt{x}+1}{\sqrt{x}-1}$

We can insert numbers for the variables:

Ex $\frac{2y}{x^2+1}$ with $x = 3$, $y=-1$ gives a

number: $\frac{2\cdot(-1)}{3^2+1} = \frac{-2}{9+1} = \frac{-2}{10} = \frac{-1}{5} = -0.2$

– but $\frac{2y}{x^2+1}$ cannot be simplified.

Problem We have the rational expression $\frac{x^2-x-6}{x-3}$.

a) Fill in

x	1	5	-2	2	8	3
$\frac{x^2-x-6}{x-3}$	3	7	0	4	10	"$\frac{0}{0}$" undefined

b) Find the pattern. Add two to the x-value (except $x=3$)

shorter (w. alg.): $x+2$ $(x \neq 3)$

Quadratic expansion

$$(x+r)^2 = (x+r)\cdot(x+r) = x^2 + 2rx + r^2$$

Ex $(x+5)^2 = x^2 + 10x + 25$

Ex $13^2 = (10+3)^2 = 10^2 + 2\cdot3\cdot10 + 3^2$
$= 100 + 60 + 9 = \underline{\underline{169}}$.

Conjungate expansion

$$(x-r)(x+r) = x^2 - r^2$$

Ex $(x-5)(x+5) = x^2 - 25$

Ex $8\cdot12 = (10-2)\cdot(10+2) = 100-4 = 96$

$\boxed{\text{Start: 11.00}}$

3. Roots

Ex The square root of 5 is the positive number
a such that $a\cdot a = 5$.
(a is in the calculator: $a = 2.2361\ldots$)
We write a as $\sqrt{5}$

Note Negative numbers don't have square roots.

Ex $\sqrt{0} = 0$

Problem Compute (without calculator)

a) $(\sqrt{2}+3)^2 = (\sqrt{2})^2 + 2\cdot3\cdot\sqrt{2} + 3^2 = \underline{\underline{11+6\sqrt{2}}}$

b) $(\sqrt{5}-1)(\sqrt{5}+1) = (\sqrt{5})^2 - 1^2 = 5-1 = \underline{\underline{4}}$

Ex There are other roots.

$\sqrt[3]{5}$ is the number a such that $a \cdot a \cdot a = 5$

$\sqrt[5]{32} = 2$ since $\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ times}} = 32$.

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4. Powers - repeated multiplication

Ex $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ "three to the power of four"

$4 \cdot 4 \cdot 4 = 4^3$

exponent

$4^3 \neq 4 \cdot 3$

the base

Ex $10^2 \cdot 10^3 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5$

$= 10^{2+3}$

so $\boxed{a^n \cdot a^m = a^{n+m}}$

Ex $\dfrac{3^6}{3^4} = \dfrac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \dfrac{3 \cdot 3}{1} = 3^2 = 3^{6-4}$

(so $3^{-4} = \dfrac{1}{3^4}$)

$1 = \dfrac{5^3}{5^3} = 5^{3-3} = 5^0$

Ex $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$

$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$= 3^8 = 3^{2 \cdot 4}$

$\boxed{(a^n)^m = a^{n \cdot m}}$

5. Order of operations

Problem compute

a) $2+3\cdot 4 = \begin{cases} 5\cdot 4 = 20 & = (2+3)\cdot 4 \\ 2+12 = \underline{14} \end{cases}$

b) $2\cdot 2^4 = \begin{cases} (2\cdot 2)^4 = 256 \\ 2\cdot(2^4) = \underline{32} \end{cases}$

Problem $-5^2 = (-1)\cdot 5^2 = -25$

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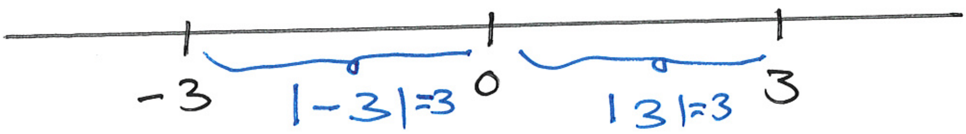
6. Absolute value

If a is a number, then $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$

"the absolute value of a"

Ex $|3| = 3$, $|-3| = -(-3) = 3$

$|a|$ = the distance between 0 and a on the number line.



Problem Simplify $\sqrt{x^2}$

Solution If $x \geq 0$ then $\sqrt{x^2} = x$

If $x < 0$ then $\sqrt{x^2} = -x$

In short: $\sqrt{x^2} = |x|$

Ex $\sqrt{(x-5)^2} = |x-5|$

Ex $\sqrt{(-3)^2} = 3 = |-3|$.