EBA1180, lecture 10, 27 Sept. 2023, Runar Ile

Plan
1. Functions and graphs
2. Linear functions and straight lines
3. Quadratic functions and parabolas
4. Revenue - and cost functions

1. Functions and graphs

Empirical functions

- the temperature as a function of time
- the price of salmon
- all kinds of 'indexes'
- fertility

A function is a table of function values

x	....
$f(x)$	.....

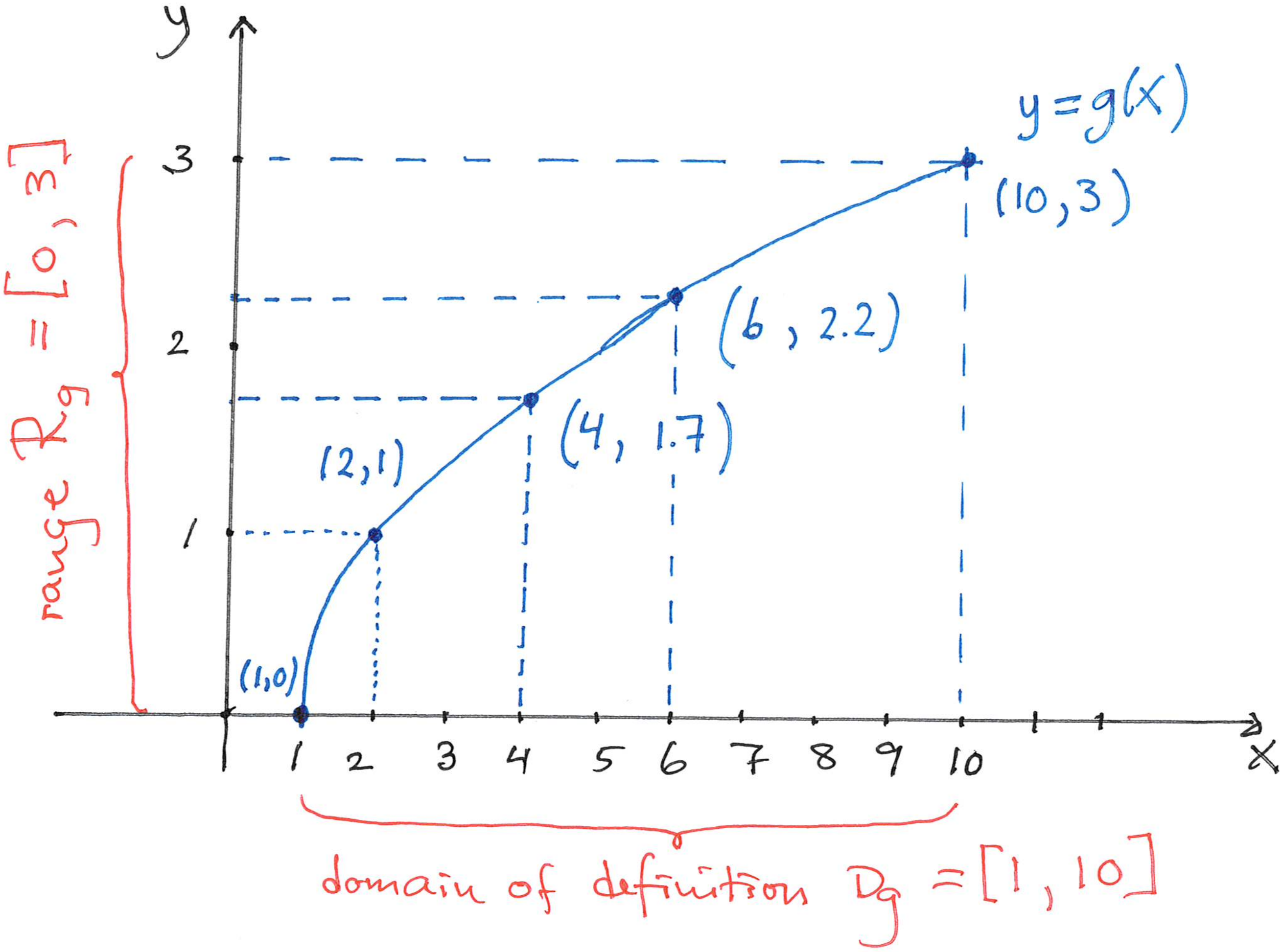
<u>Ex</u> $f(x)$ = average age at the birth of the first child in year x.

Domain of definition : $x \in [1961, 2022] = D_f$

<u>Ex</u> $g(x) = \sqrt{x-1}$. The largest possible domain is $D_g = [1, \rightarrow)$

Want to draw the graph with $D_g = [1, 10]$. I make a table of function values:

x	1	2	4	6	10
$g(x)$	0	1	1.7	2.2	3



2. <u>Linear functions</u> $\quad f(x) = ax + b$

- the graph is a line

The point-slope formula :

If (x_0, y_0) is a point on the graph (a line!) and a is the slope, then

$$y - y_0 = a \cdot (x - x_0)$$

dependent variable

independent variable

start: 11.00

Ex If $(x_0, y_0) = (9, 25)$ and $(x_1, y_1) = (11, 31)$ are two points on the line, then the slope is (the relative change)

$$a = \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}} = \frac{31 - 25}{11 - 9}$$

$$= \frac{6}{2} = 3$$

The point-slope formula gives

$$y - 25 = 3 \cdot (x - 9)$$

so $y = 3x - 27 + 25$

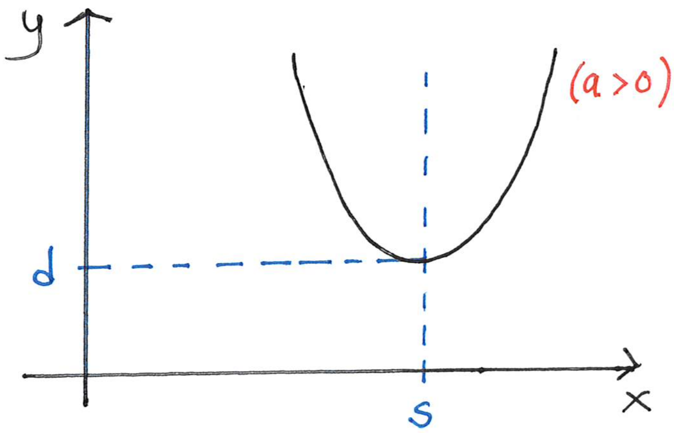
so $\underline{\underline{y = 3x - 2}}$

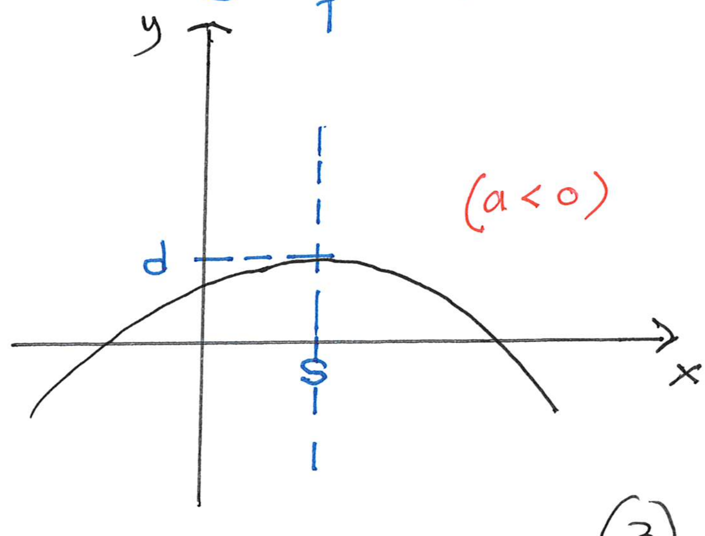
3. Quadratic functions

$$f(x) = ax^2 + bx + c$$

But if we want to draw/understand the graph, the following standard expression is better:

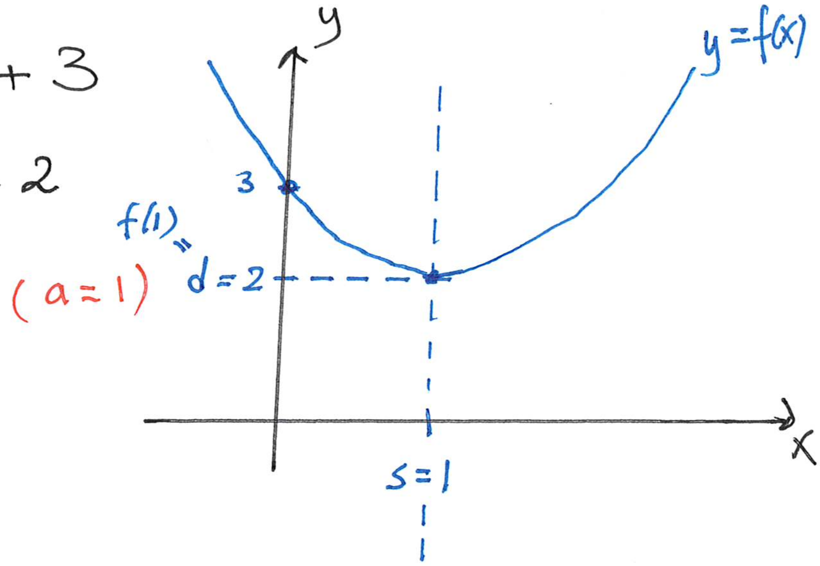
$$f(x) = a(x - s)^2 + d$$ "by completing the square"





Ex $f(x) = x^2 - 2x + 3$

$= (x-1)^2 + 2$



Problem

The quadratic function $f(x)$ has minimum value $y = -1$ and symmetry line $x = 5$ and the graph passes through the point $(9, 3)$.

a) Determine the expression $f(x) = a(x-s)^2 + d$

b) Determine where the graph of $f(x)$ crosses the x-axis and the y-axis.

Solution

a) We have been given $s = 5$ and $d = -1$.

so $f(x) = a(x-5)^2 - 1$

Then $f(9) = 3$ gives $a(9-5)^2 - 1 = 3$

$a \cdot 16 = 4$

$a = \frac{4}{16} = 0.25$

and $f(x) = 0.25(x-5)^2 - 1$

b) Crosses x-axis: Solve $f(x) = 0$

i.e. $0.25(x-5)^2 - 1 = 0$ $\quad 1 \cdot 4$

get $(x-5)^2 = 4$

so $x - 5 = \pm 2$ so $x = 3$ or $x = 7$

Crosses the y-axis: $y = f(0) = 0.25 \cdot (0-5)^2 - 1$

$= 0.25 \cdot 25 - 1 = 6.25 - 1 = \underline{\underline{5.25}}$

or as point: $\underline{\underline{(0, 5.25)}}$

4. Revenue and cost functions

Profit = Revenue − cost

$P(x) = R(x) - C(x)$

x = number of units produced

= units sold

Ex $R(x) = 15x$, $C(x) = 0.05x^2 - 10x + 525$

Determine the number of units x which maximizes profit. calculate maximal profit.

$P(x) = 15x - (0.05x^2 - 10x + 525)$

complete the sq.

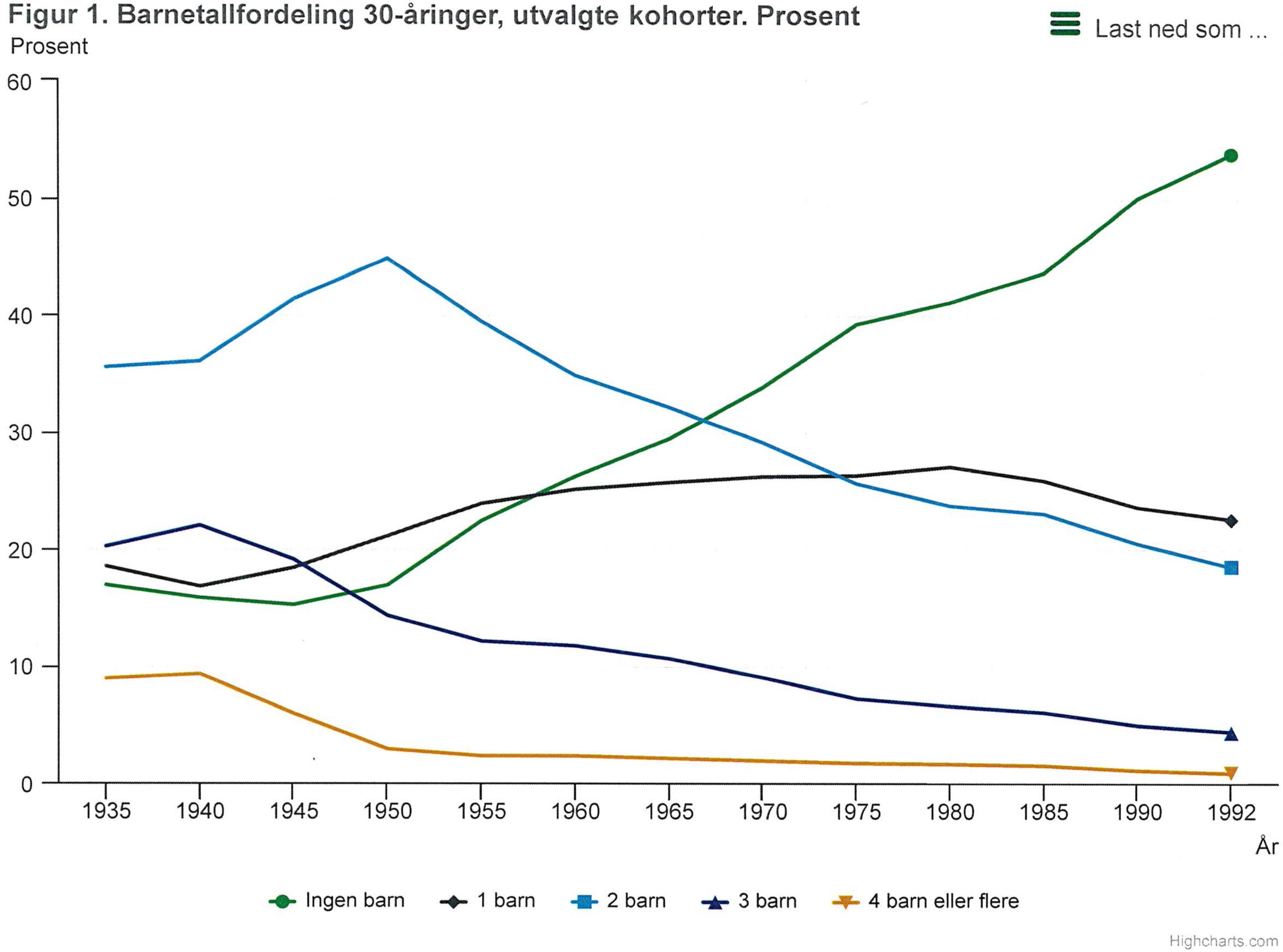
$= -0.05\left[(x-250)^2 - 52000\right]$

So max. profit if $\underline{\underline{x = 250}}$

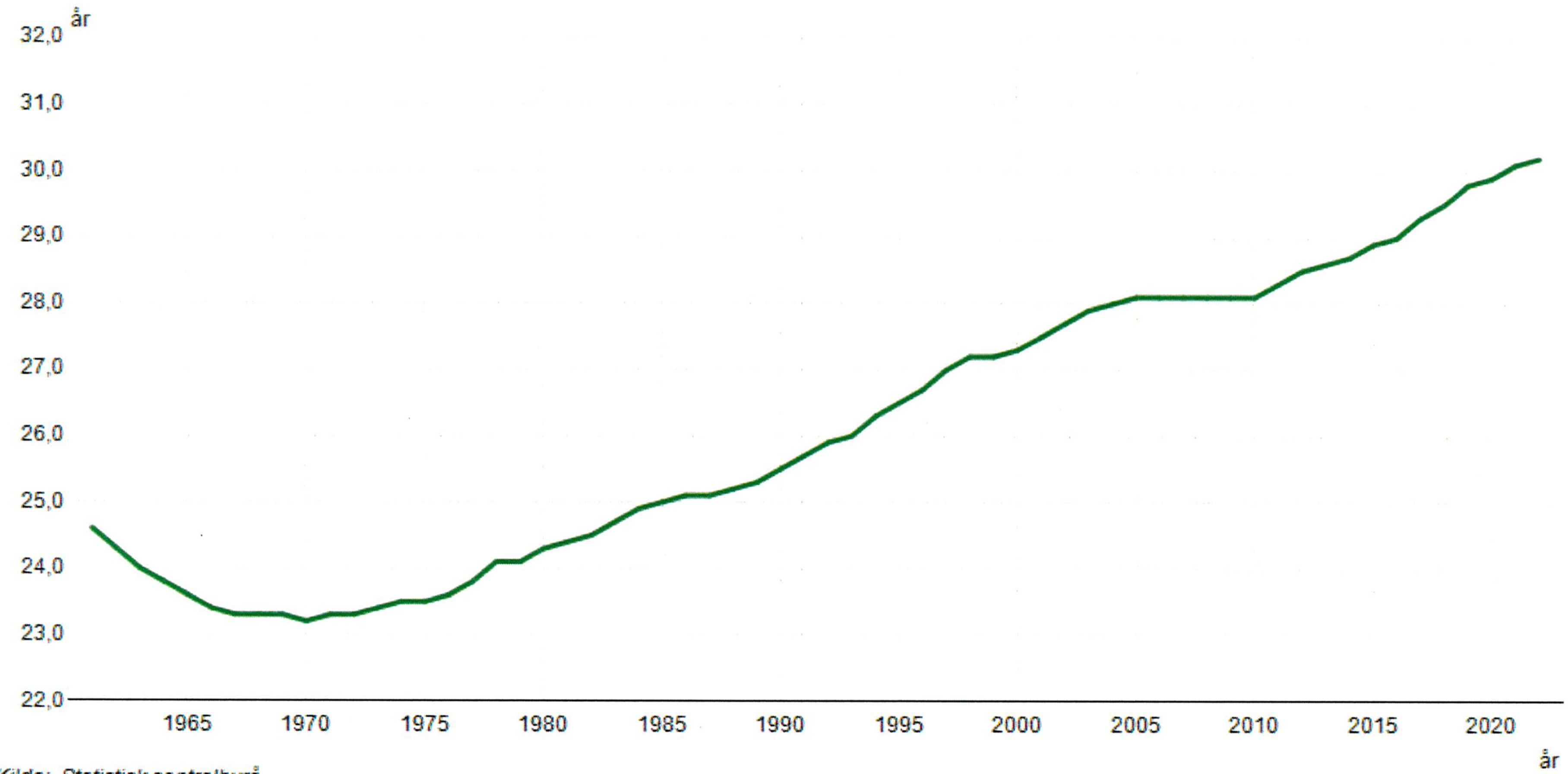
Max. profit $= P(250) = -0.05 \cdot (-52000)$

$= \underline{\underline{2600}}$

Figur 1. Barnetallfordeling 30-åringer, utvalgte kohorter. Prosent



07872: Foreldrenes gjennomsnittlige fødealder ved første barns fødsel, etter år. Mors fødealder første barn.

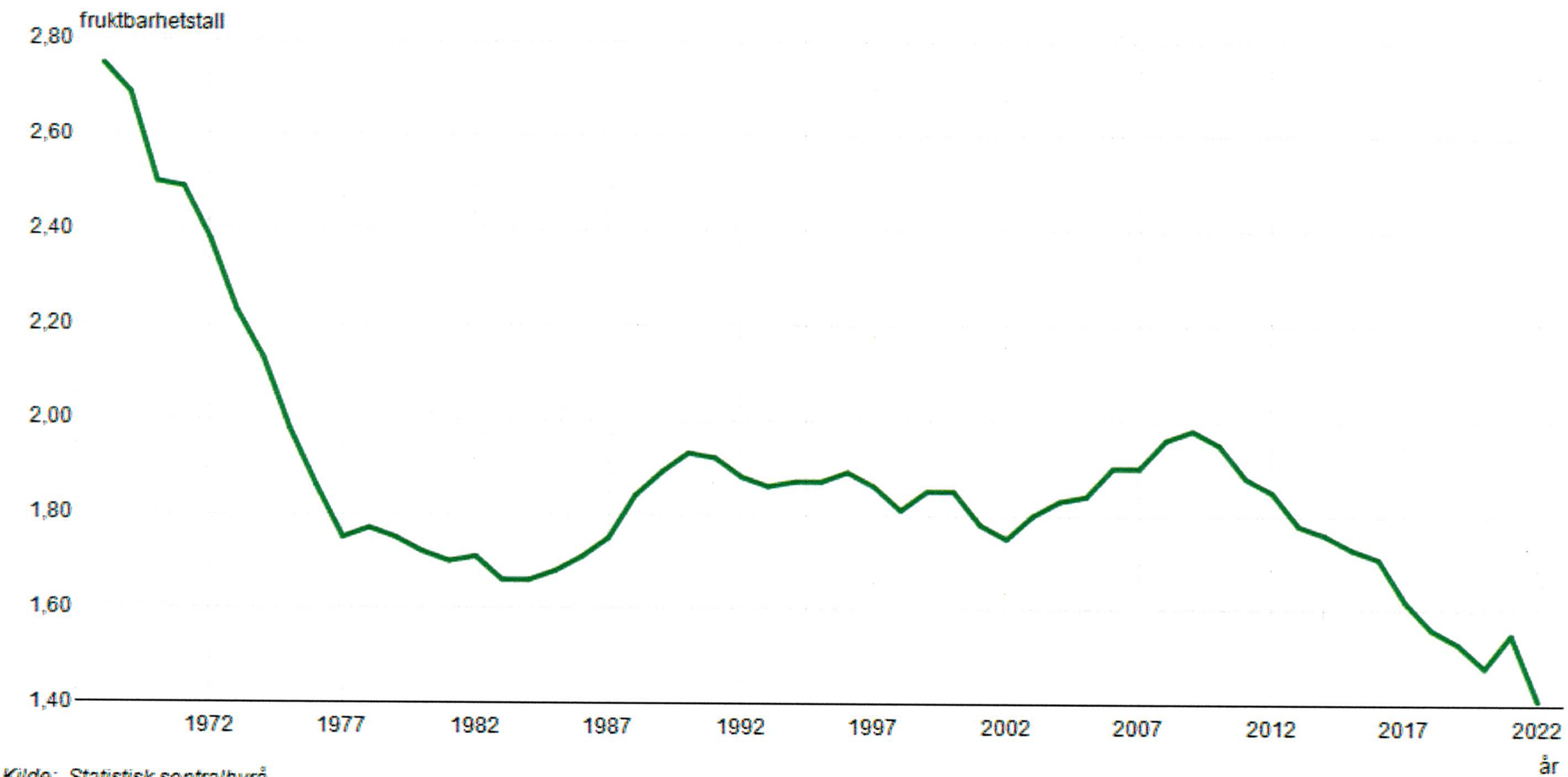


Kilde: Statistisk sentralbyrå

Fotnoter

Tall for 1961-1985 er beregnet ut fra nytt tilgjengelig datagrunnlag fra 2009. Tilsvarende datagrunnlag brukes for beregning av fars gjennomsnittsalder ved første barns fødsel.

04232: Samlet fruktbarhetstall, kvinner, etter år. Samlet fruktbarhetstall, kvinner.



Kilde: Statistisk sentralbyrå

Fotnoter

Samlet fruktbarhetstall er summen av 1-årige aldersavhengige fruktbarhetsrater 15-49 år. Antall barn hver kvinne kommer til å føde under forutsetning av at fruktbarhetsmønstret i perioden varer ved og at dødsfall ikke forekommer.