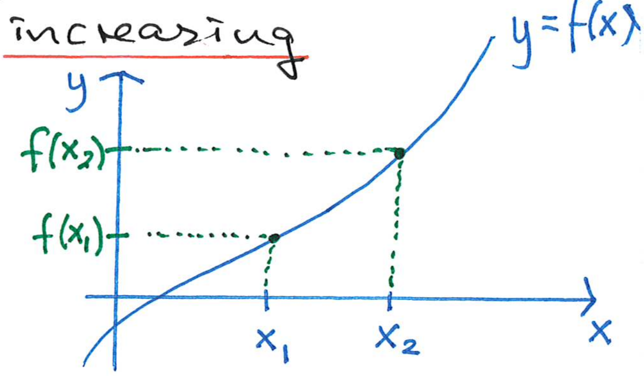
EBA 1180, lecture 12, 4 Oct. 2023, Runar Ile

Plan
1. Increasing and decreasing functions
2. Circles and ellipses
3. Polynomial functions

1. Increasing and decreasing functions

Definition A function $f(x)$ is increasing if for all $x_1 < x_2$ one has $f(x_1) \leq f(x_2)$



Ex $f(x) = 2x+5$ is increasing for all x.

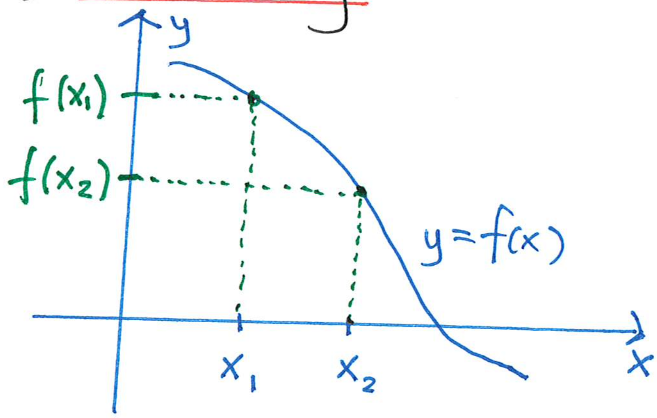
Reason: Assume $x_1 < x_2 \quad | \cdot 2$

$$2x_1 < 2x_2 \quad | +5$$

$$f(x_1) = 2x_1+5 < 2x_2+5 = f(x_2)$$

so $f(x)$ is increasing (actually strictly increasing)

Definition A function $f(x)$ is decreasing if for all $x_1 < x_2$ one has $f(x_1) \geq f(x_2)$



Problem Show that $f(x) = -2x+5$ is (strictly) decreasing.

Solution suppose $x_1 < x_2 \quad | \cdot (-2)$

$$-2x_1 > -2x_2 \quad | +5$$

$$f(x_1) = -2x_1 + 5 > -2x_2 + 5 = f(x_2)$$

Problem We have a constant function $f(x) = 5$. Decide whether $f(x)$ is increasing, decreasing or neither.

Solution

Increasing: If $x_1 < x_2$ then $f(x_1) = 5 \leq 5 = f(x_2)$

Decreasing: If $x_1 < x_2$ then $f(x_1) = 5 \geq 5 = f(x_2)$

– so both. But neither strictly increasing nor —"— decreasing.

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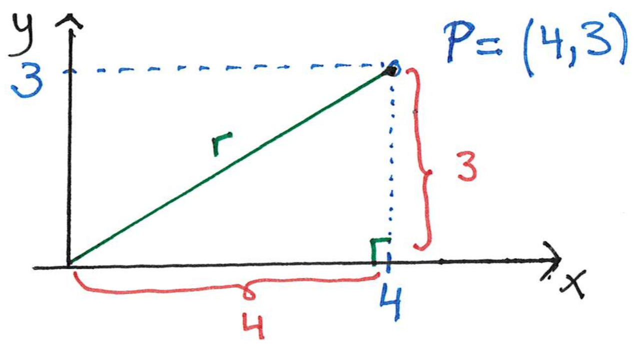
2. Circles and ellipses

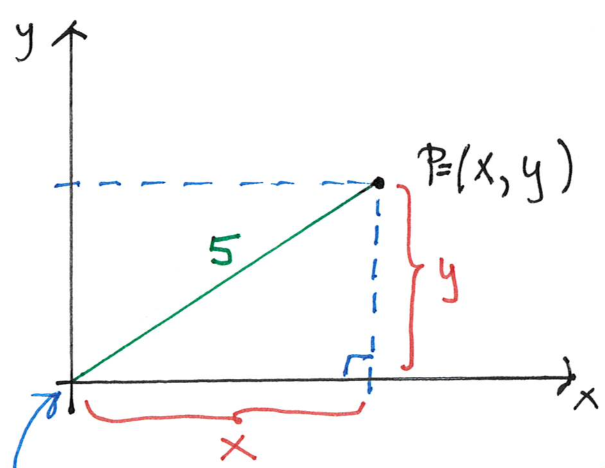


Pytagoras:

$$r^2 = 4^2 + 3^2 \quad (r \geq 0)$$

$$r^2 = 16 + 9 = 25$$

$$r = \sqrt{25} = 5$$

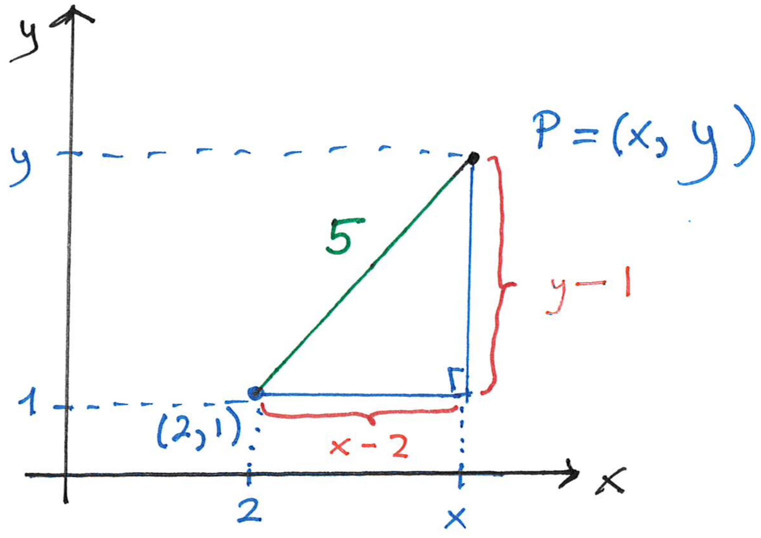


Pytagoras:

$$25 = x^2 + y^2$$

- one equation
- two unknowns
- infinitly many solutions

origin (0,0) is the centre of the circle of solutions, with radius = 5



Pytagoras:

$$5^2 = (x-2)^2 + (y-1)^2$$

$$25 = x^2 - 4x + 4 + y^2 - 2y + 1$$

that is

$$x^2 + y^2 - 4x - 2y = 20$$

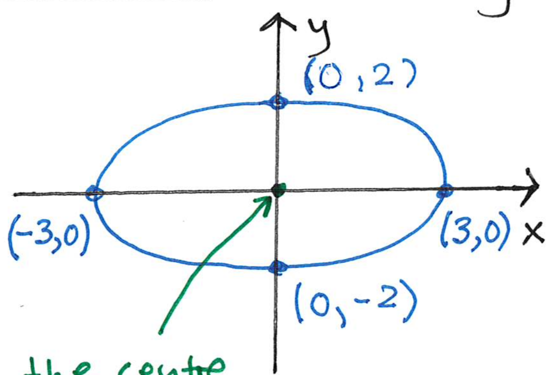
<u>Problem</u> Determine the radius and the centre of $x^2 + y^2 - 2x + 6y = -9$

<u>Solution</u> $\underbrace{(x-1)^2}_{x^2-2x+1} + \underbrace{(y+3)^2}_{y^2+6y+9} = -9 + 1 + 9 = 1$

Centre : $\underline{\underline{(1, -3)}}$, radius : $\sqrt{1} = \underline{\underline{1}}$

<u>Ellipses</u> $4x^2 + 9y^2 = 36$

x	3	-3	0	0
y	0	0	2	-2



the centre of the ellipse : (0, 0)

I divide each side of the equation by 36 and get

$$\frac{4}{36}x^2 + \frac{9}{36}y^2 = 1 \qquad \frac{4}{36} = \frac{1}{9}, \quad \frac{9}{36} = \frac{1}{4}$$

Similar to a circle equation

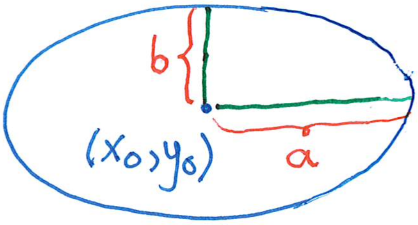
$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

but the x-axis is stretched by a factor 3 and the y-axis is stretched by a factor 2.

In general, any ellipse is the set of solutions of an equation of the form

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

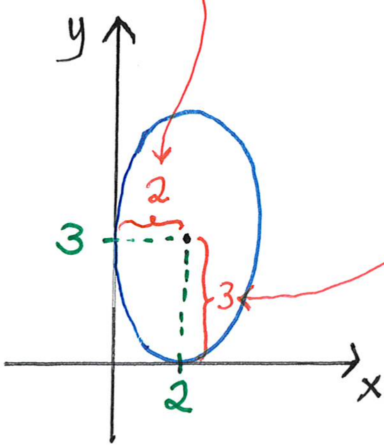
Here (x_0, y_0) is the centre and a and b are semi-axes



Ex $\quad \dfrac{(x-2)^2}{4} + \dfrac{(y-3)^2}{9} = 1$

Centre: $(2, 3)$

Semi-axes: $a = \sqrt{4} = 2, \quad b = \sqrt{9} = 3$



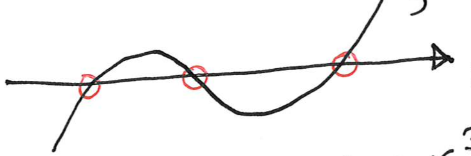
3. Polynomial functions

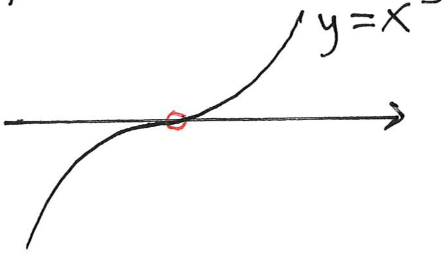
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad (a_n \neq 0)$

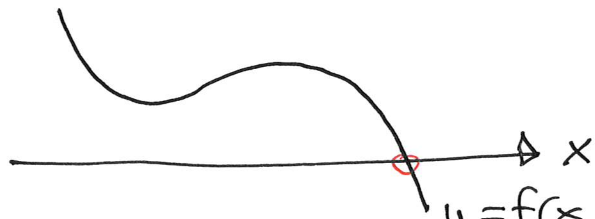
has degree n, write $\deg(f) = n$

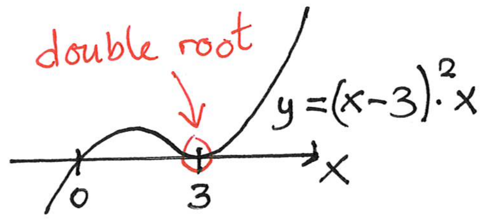
Then:
- $f(x)$ has at most n roots (zeros)
- If the degree is a odd number, then $f(x)$ has at least one zero.
- If a polynomial $h(x)$ has m roots then $\deg(h) \geq m$

EX

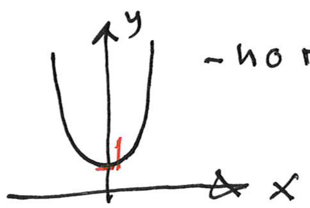




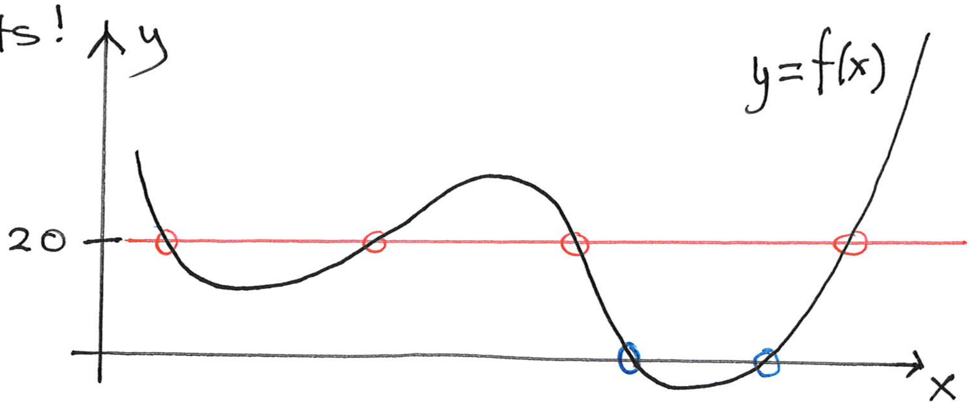




EX $f(x) = x^4 + 1$



– no roots!



$f(x) = 0$ has only 2 roots, but

$f(x) = 20$ has 4 solutions

that is $\underbrace{f(x) - 20}_{} = 0$ has the same 4 solutions (roots!)

– a polynomial of the same degree as $f(x)$.

So $f(x)$ has at least degree 4.