EBA1180
Spring 24
Sect. 1

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- Info: Lecture plan, lecture notes, exercise sheets, messages: It's learning (dr.eriksen.no)

Exams: Final exam + retake exams

Topics

→ Integration

→ Matrix and vector computations

→ Functions in two variables + everything from last semester

NEW TOPIC: Integration

Definite integrals

Def: (Definite integral)

$$\int_a^b f(x)\,dx = \text{area of the region between the graph of } f \text{ and the x-axis in } [a,b]$$

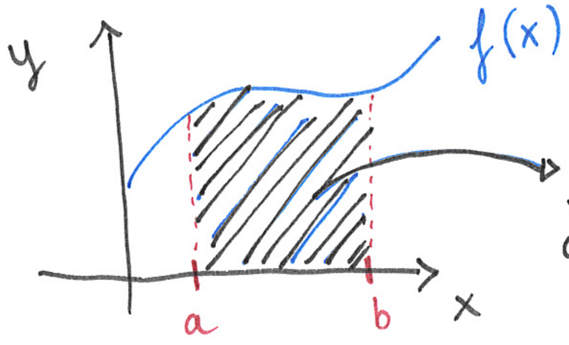
integration sign

function we're integrating

b → integration bounds: upper bound

a integration bounds: lower bound

Read: "Integral from a to b of $f(x)\,dx$"




$$\int_a^b f(x)\,dx$$

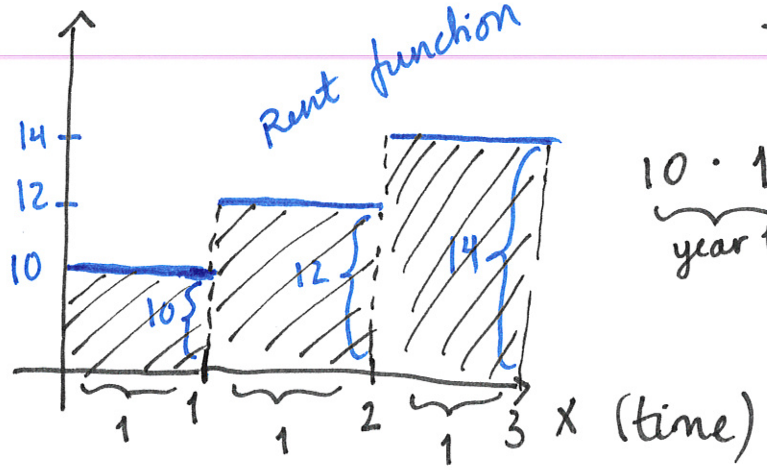
ASSUMPTIONS: i) $f(x)$ is a continuous function on $[a, b]$

ii) $f(x) \geq 0$ on $[a, b]$

iii) $a < b$

Ex: Rental income over 3 years

Rent changed per year




Total rent income:

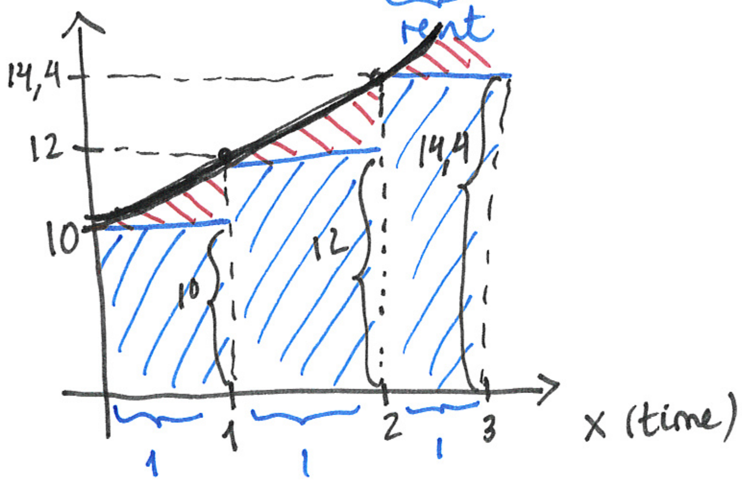
$$\underbrace{10 \cdot 1}_{\text{year 1}} + \underbrace{12 \cdot 1}_{\text{year 2}} + \underbrace{14 \cdot 1}_{\text{year 3}}$$

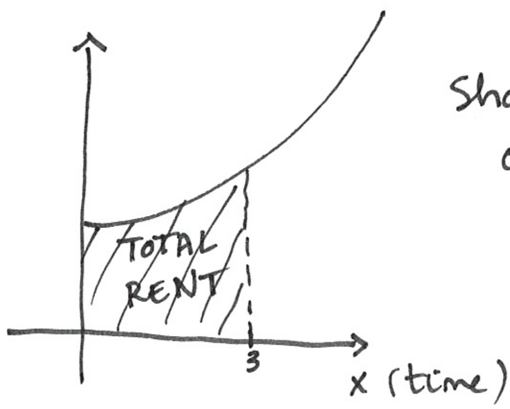
$$= 36$$

= area of the rectangles under graph of rent-function

Now: Assume we have a continuously changing rent:

$$\underbrace{f(x)}_{\text{rent}} = 10 \cdot 1{,}2^{x}$$




Shape of curve

Total rent income over 3 years

$$= \int_0^3 10 \cdot 1{,}2^x \, dx = \text{area under the graph between 0 and 3}$$

$$f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1$$

$$\approx 10\cdot 1 + 12 \cdot 1 + 14{,}4 \cdot 1 = 36{,}4$$

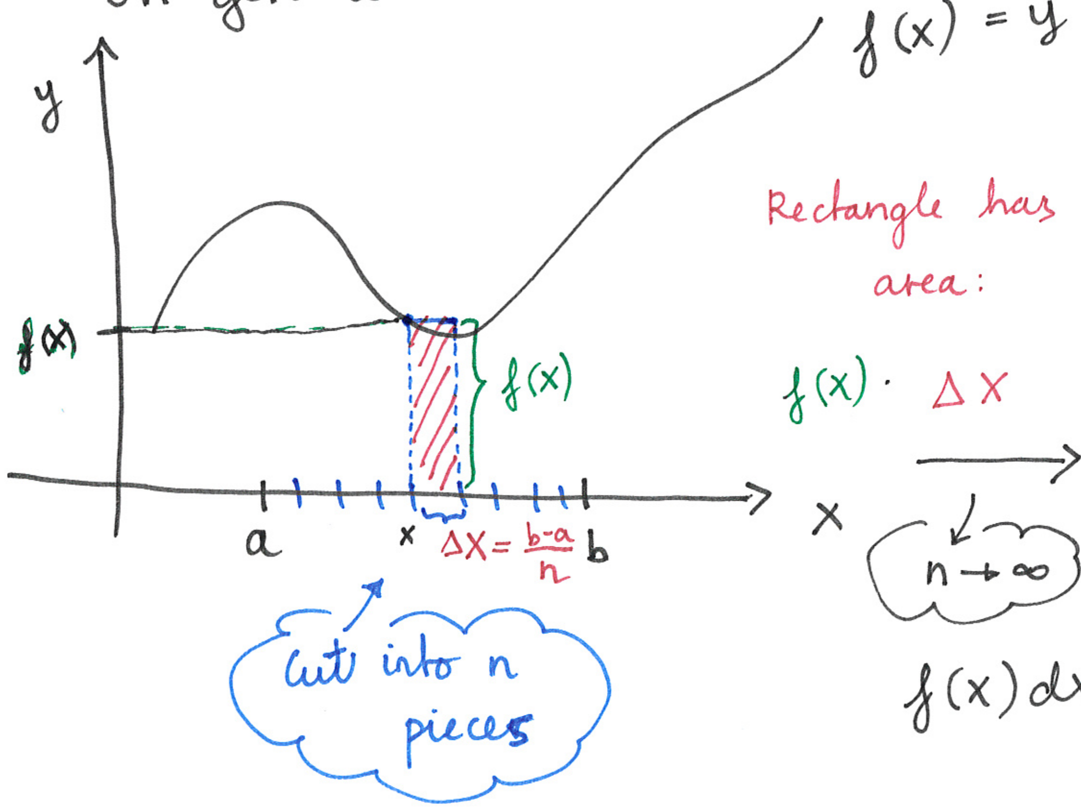
Riemann sum

NB: Approximation! Missing some bits.

Exercise: How to make approximation better?

Idea: Make width/increments smaller.

NOTE: In general




Rectangle has area:

$f(x) \cdot \Delta x \xrightarrow{n \to \infty} f(x)\,dx$

# Anti-derivatives and indefinite integrals

Def: (Anti-derivative)

An antiderivative of a function $f(x)$ is a function $F(x)$ s.t. $F'(x) = f(x)$.

Ex: $f(x) = 2x \Rightarrow F(x) = x^2$

because $F'(x) = (x^2)' = 2x = f(x)$

Q: Can you think of another antiderivative of $f(x)$?

Say: $F(x) = x^2 + 1$, because $F'(x) = (x^2+1)'$
$= 2x \ (= f(x))$

Or more generally: $F(x) = x^2 + C$, because
$F'(x) = (x^2 + C)' = 2x (= f(x))$

FACT: If $f(x)$ has an antiderivative $F(x)$, then any other antiderivative can be written as $F(x) + C$ where $C$ is a constant.

## Def (Indefinite integral):

The indefinite integral of a function $f(x)$ is

$$\int f(x)\,dx = \underbrace{F(x) + C}_{\text{All antiderivatives of } f(x)}$$

where $F'(x) = f(x)$.

Ex: $$\int 2x\,dx = x^2 + \underbrace{C}_{\text{integration constant}}$$

## Integration rules

$\boxed{\text{How to compute } \int f(x)\,dx \text{ ?}}$ ⤳ ANTI-DIFFERENTIATE

Ex: $$\int 3 + x + x^2\,dx = 3x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$$

CHECK: Differentiate the answer:

$$\left(3x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + C\right)' = 3 + \frac{1}{2}\cdot 2x + \frac{1}{3}3x^2$$

$$= 3 + x + x^2$$

THE SAME!

# INTEGRATION RULES

i) Power rule: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

CHECK: $\left(\frac{x^{n+1}}{n+1} + C\right)' = \frac{\cancel{(n+1)} x^{n+1-1}}{\cancel{n+1}} + 0 = x^n$

ii) $\int \frac{1}{x} \, dx = \ln|x| + C$

Integral of sum is the sum of the integrals:

iii) $\int u(x) + v(x) \, dx$

$= \int u(x) \, dx + \int v(x) \, dx$

Multiplied constants can be moved outside the integral:

iv) $\int c \cdot u(x) = c \int u(x) \, dx \quad$ (c is a constant)

v) Exponentials: $\int e^x \, dx = e^x + C$

$(e^x)' = e^x$

$\int a^x \, dx = \frac{1}{\ln(a)} a^x + C \quad (a > 0)$

$(a^x)' = a^x \ln(a)$

NOTE: Need to add integration constant when solving indefinite integral.

Ex: $\int 3x^5 \, dx \underset{(iv)}{=} 3 \int x^5 \, dx \underset{\text{Power rule}}{=} 3 \cdot \frac{x^{5+1}}{5+1} + C$

$= 3 \frac{x^6}{6} + C$

$= \frac{1}{2} x^6 + C$

Exercise: $\int 3x^5 + 6x^{12}\,dx$

$$\overset{iii)}{=} \int 3x^5\,dx + \int 6x^{12}\,dx$$

$$\overset{ii)}{=} \frac{1}{2}x^6 + 6\,\frac{x^{13}}{13} + C$$

+ prev. ex.
+ i)

$$= \frac{1}{2}x^6 + \frac{6}{13}x^{13} + C$$