Exercise session problems

Problem 1.
Compute the following inner products when

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad \vec{v}_4 = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$$

a) $\vec{v}_1 \cdot \vec{v}_2$
b) $\vec{v}_1 \cdot \vec{v}_3$
c) $\vec{v}_2 \cdot \vec{v}_2$
d) $(\vec{v}_1 - \vec{v}_2) \cdot \vec{v}_4$
e) $\vec{v}_1 \cdot (\vec{v}_2 + \vec{v}_3)$
f) $\vec{v}_1 \cdot (\vec{v}_2 - \vec{v}_3)$
g) $(\vec{v}_4 - \vec{v}_1) \cdot (\vec{v}_2 + \vec{v}_3)$
h) $(\vec{v}_1 - \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2)$

Problem 2.
Find all the vectors that are orthogonal to the vector $\mathbf{v}$:

a) $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$
b) $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
c) $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$
d) $\mathbf{v} = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$

Problem 3.
Determine $||\vec{v} - \vec{w}||$ when the vectors $\vec{v}, \vec{w}$ are orthogonal and have length $||\vec{v}|| = 3$ and $||\vec{w}|| = 4$.

Problem 4.
Find the natural domain D_f and range V_f of the function f:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = \sqrt{x + 3y}$
c) $f(x,y) = (2x - y)^{-3/2}$
d) $f(x,y) = 17x^{1.2}y^{3.4}$

Problem 5.
Consider the level curve $f(x,y) = c$ of a function $f(x,y)$. Draw the level curves for the given values of c in the same coordinate system and determine what kind of curve we get when we let c be an arbitrary value:

a) $f(x,y) = 12x - 3y$ og $c = -3, 0, 3$
b) $f(x,y) = xy$ og $c = -1, 0, 1$
c) $f(x,y) = x^2 + 2x + y^2 - 4y$ og $c = -9, -5, -1$
d) $f(x,y) = x^2 - 2x + 4y^2$ og $c = -2, -1, 0, 1$

Problem 6.
Use the level curves $f(x,y) = c$ from Problem 5 to determine whether the following functions have maximum- or minimum values:

a) $f(x,y) = 12x - 3y$
b) $f(x,y) = xy$
c) $f(x,y) = x^2 + 2x + y^2 - 4y$
d) $f(x,y) = x^2 - 2x + 4y^2$

Problem 7.

Describe the graph of $f(x,y) = 3x - 4y + 1$ geometrically. What is meant by a geometrical description is for example: *The graph of $f(x) = 3 - 2x$ is a straight line with slope -2 which intersects the y-axis in $y = 3$*, i.e., a precise geometrical description without using equations etc.

Problem 8.

Find the partial derivatives f'_x and f'_y when

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 + y^2$
c) $f(x,y) = 4x^2 - 6xy + 9y^2$
d) $f(x,y) = x^2 - 2x + 4y^2$
e) $f(x,y) = x^3 - 3xy + y^3$
f) $f(x,y) = y^2 - x^3 + 3x$
g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$
h) $f(x,y) = \sqrt{x^2 + y^2}$

Problem 9.

Find the Hessian matrix $H(f)$, and compute $H(f)(1,1)$:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 + y^2$
c) $f(x,y) = 4x^2 - 6xy + 9y^2$
d) $f(x,y) = x^2 - 2x + 4y^2$
e) $f(x,y) = x^3 - 3xy + y^3$
f) $f(x,y) = y^2 - x^3 + 3x$
g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$
h) $f(x,y) = \sqrt{x^2 + y^2}$

Problem 10.

Find the stationary points of f, and classify them:

a) $f(x,y) = 2x + 3y$
b) $f(x,y) = x^2 + y^2$
c) $f(x,y) = 4x^2 - 6xy + 9y^2$
d) $f(x,y) = x^2 - 2x + 4y^2$
e) $f(x,y) = x^3 - 3xy + y^3$
f) $f(x,y) = y^2 - x^3 + 3x$
g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$
h) $f(x,y) = \sqrt{x^2 + y^2}$

Problem 11.

Find all stationary points and classify them:

a) $f(x,y) = xy(x^2 - y^2)$
b) $f(x,y) = x^2y + xy^3 + xy^2$
c) $f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$

Oppgaver fra læreboken

Læreboken [E]: Eriksen, *Matematikk for økonomi og finans*
Oppgaveboken [O]: Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Oppgaver:	[E] 7.1.1 - 7.1.4, 7.2.1 - 7.2.2, 7.3.1 - 7.3.5, 7.4.1 - 7.4.2
Fullstendig løsning:	Se [O] Kap 7.1 - 7.4

Answers to the exercise session problems

Problem 1.

a) 0 b) 4 c) 2 d) 13

e) 4 f) -4 g) 18 h) 5

Problem 2.

All linear combinations of the following vectors:

a) $\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\0\end{pmatrix}$ b) $\begin{pmatrix}-1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}$ c) $\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}-2\\1\\0\end{pmatrix}$ d) $\begin{pmatrix}3\\0\\4\end{pmatrix}, \begin{pmatrix}-7\\4\\0\end{pmatrix}$

Problem 3.

5

Problem 4.

a) $D_f = \mathbb{R}^2,\ V_f = \mathbb{R}$

b) $D_f = \{(x,y) \in \mathbb{R}^2 : x + 3y \geq 0\},\ V_f = [0,\infty)$

c) $D_f = \{(x,y) \in \mathbb{R}^2 : 2x - y > 0\},\ V_f = (0,\infty)$

d) $D_f = \{(x,y) \in \mathbb{R}^2 : x,y \geq 0\},\ V_f = [0,\infty)$

Problem 5.

a) Straight line with slope 4 which intersects the y-axis in $y = c/3$

b) Hyperbola $y = c/x$ if $c \neq 0$, and the two axis if $c = 0$

c) Circle with radius $\sqrt{c+5}$ and center $(-1,2)$ if $c > -5$, one point $(-1,2)$ if $c = -5$, no points otherwise.

d) Ellipse with center in $(1,0)$ with half axis $a = \sqrt{c+1}$ and $b = \sqrt{c+1}/2$ when $c > -1$, one point $(1,0)$ if $c = -1$, and no points otherwise.

Problem 6.

a) Neither maximum nor minimum.

b) Neither maximum nor minimum.

c) No maximum, but the minimum value is $f_{min} = -5$.

d) No maximum, but the minimum value is $f_{min} = -1$

Problem 7.

The graph is the plane which intersects the z-axis in $z = 1$ and has the normal vector $(3, -4, -1)$.

Problem 8.

a) $f'_x = 2,\ f'_y = 3$

b) $f'_x = 2x,\ f'_y = 2y$

c) $f'_x = 8x - 6y,\ f'_y = -6x + 18y$

d) $f'_x = 2x - 2,\ f'_y = 8y$

e) $f'_x = 3x^2 - 3y,\ f'_y = -3x + 3y^2$

f) $f'_x = -3x^2 + 3,\ f'_y = 2y$

g) $f'_x = 2x(y^2 - 1),\ f'_y = 2y(x^2 - 1)$

h) $f'_x = \dfrac{x}{\sqrt{x^2+y^2}},\ f'_y = \dfrac{y}{\sqrt{x^2+y^2}}$

Problem 9.

a) $H(f) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

b) $H(f) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

c) $H(f) = \begin{pmatrix} 8 & -6 \\ -6 & 18 \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} 8 & -6 \\ -6 & 18 \end{pmatrix}$

d) $H(f) = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$

e) $H(f) = \begin{pmatrix} 6x & -3 \\ -3 & 6y \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix}$

f) $H(f) = \begin{pmatrix} -6x & 0 \\ 0 & 2 \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} -6 & 0 \\ 0 & 2 \end{pmatrix}$

g) $H(f) = \begin{pmatrix} 2(y^2-1) & 4xy \\ 4xy & 2(x^2-1) \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$

h) $H(f) = (x^2+y^2)^{-3/2} \cdot \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}$, $H(f)(1{,}1) = \begin{pmatrix} \sqrt{2}/4 & -\sqrt{2}/4 \\ -\sqrt{2}/4 & \sqrt{2}/4 \end{pmatrix}$

Problem 10.

a) None

b) (0,0) is a local min.

c) (0,0) is a local min min.

d) (1,0) is a local min min.

e) (0,0) is a saddle point and (1,1) is a local min.

f) (1,0) is a saddle point and $(-1{,}0)$ is a local min.

g) (0,0) is a local max and $(\pm 1, \pm 1)$ is a saddle point.

h) none; (0,0) is a critical point.

Problem 11.

a) (0,0) is a saddle point.

b) (0,0), $(0, -1)$ is a saddle point, $(3/25, -3/5)$ is a local max.

c) (0,0) is a local (and global) max.