Exercise session problems

Problem 1.
Use the Lagrange multiplier method to find candidates for the maximum and/or minimum:

a) $\max / \min f(x,y) = 3x - y$ when $x^2 + 4y^2 = 37$
b) $\max / \min f(x,y) = x^2 + 4y^2$ when $3x - y = 37$
c) $\max / \min f(x,y) = xy$ when $x^2 + 4y^2 = 8$
d) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $xy = 6$
e) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $x^2 + y^2 = 16$
f) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $xy = 4$

Problem 2.
Find the maximum/minimum, if it exists:

a) $\max / \min f(x,y) = 3x - y$ when $x^2 + 4y^2 = 37$
b) $\max / \min f(x,y) = x^2 + 4y^2$ when $3x - y = 37$
c) $\max / \min f(x,y) = xy$ when $x^2 + 4y^2 = 8$
d) $\max / \min f(x,y) = 4x^2 + 9y^2$ when $xy = 6$
e) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $x^2 + y^2 = 16$
f) $\max f(x,y) = x^2y^2 - x^2 - y^2 + 16$ when $xy = 4$

Problem 3.
Solve the Lagrange problem: $\max U(x,y) = 0.3 \ln(x - 3) + 0.7 \ln(y - 2)$ when $12x + 5y = 60$.

Problem 4.
Exam MET1180 (December 2015) Exercise 5
Consider the level curve $g(x,y) = 0$, where g is the function $g(x,y) = x^3 + xy + y^2$.

a) Find all points on the level curve with $x = -2$, and determine the tangent in each of these points.
b) Find the maximum value of $f(x,y) = x$ under the constraint $x^3 + xy + y^2 = 0$.

Problem 5.
Exam MET1180 (June 2016) Exercise 5
Consider the Lagrange problem $\max / \min f(x,y) = x + 2y - \sqrt{36 - x^2 - 4y^2}$ when $x^2 + 4y^2 = 36$.

a) Find the points on the level curve $x^2 + 4y^2 = 36$ where the tangent has slope $y' = 1/2$.
b) Make a sketch of $D = \{(x,y) : x^2 + 4y^2 = 36\}$. Is D bounded? What kind of curve is this?
c) Solve the Lagrange problem and find the maximum- and minimum value.
d) Solve the new optimization problem we get when we change the constraint to $x^2 + 4y^2 \leq 36$.

Problem 6.
Difficult!
Solve the Lagrange problem $\max f(x,y) = x + y$ when $x^3 - 3xy + y^3 = 0$. You can assume that the problem has a maximum.

Textbook [E]:	Eriksen, *Matematikk for økonomi og finans*
Exercise book [O]:	Eriksen, *Matematikk for økonomi og finans - Oppgaver og Løsningsforslag*

Exercises:	[E] 7.6.3 - 7.6.6
Solution manual:	See [O] Ch. 7.6

Optional: Exercises from the Norwegian textbook

Answers to the exercise session problems

Problem 1.

a) $(x,y;\lambda) = (6,-1/2;1/4),\ (-6,1/2;-1/4)$

b) $(x,y;\lambda) = (12,-1;8)$

c) $(x,y;\lambda) = (2,1;1/4),\ (-2,-1;1/4),\ (2,-1;-1/4),\ (-2,1;-1/4)$

d) $(x,y;\lambda) = (3,2;12),\ (-3,-2;12)$

e) $(x,y;\lambda) = (\pm 2\sqrt{2},\pm 2\sqrt{2};7), (\pm 4,0;-1), (0,\pm 4;-1)$

f) $(x,y;\lambda) = (2,2;-2), (-2,-2;-2)$

Problem 2.

a) $f_{\max} = 37/2,\ f_{\min} = -37/2$

b) $f_{\min} = 148$ (does not have a maximum)

c) $f_{\max} = 2,\ f_{\min} = -2$

d) $f_{\min} = 72$ (does not have a maximum)

e) $f_{\max} = 64,\ f_{\min} = 0$

f) $f_{\max} = 24$ (does not have a minimum)

Problem 3.

We find the maximum point $(x,y) = (67/20, 99/25)$, maximum value $f_{\max} = 1.7\ln(1.4) - 0.6\ln(2)$ with $\lambda = 1/14$.

Problem 4.

a) $y = -8x/3 - 4/3$ i $(-2,4)$ and $y = 5x/3 + 4/3$ i $(-2,-2)$

b) $f_{\max} = 1/4$

Problem 5.

a) $(3\sqrt{2},-3\sqrt{2}/2), (-3\sqrt{2},3\sqrt{2}/2)$

b) Yes, ellipse with half axes $a = 6$ and $b = 3$ with center $(0,0)$

c) $f_{\max} = 6\sqrt{2},\ f_{\min} = -6\sqrt{2}$

d) $f_{\max} = 6\sqrt{2},\ f_{\min} = -6\sqrt{3}$

Problem 6.

$f_{\max} = 3$