EBA 1180, lecture 1, 21 Aug. 2024, Runar Ile

Plan

1. Intro. to the Course
2. Algebraic expressions
3. Roots
4. Powers
5. Order of operations
6. Absolute value

1. Intro. to the Course

Autumn (40% exam)

- Financial math.
- Functions and graphs
- Differentiation and optimization

Spring (60% exam)

- Integration
- Systems of lin. eq.
- Functions in two variables $z = f(x, y)$.

2. Algebraic expressions

Variables: $x, y, z, x_1, x_2, x_3, \ldots$

$a, b, c, \ldots, m, n \ldots$

Multiply with a number:

$$3 \cdot x \overset{\text{short form}}{=} 3x$$

$$3 \cdot 2 \neq 32$$

$$\sqrt{3} \cdot x = \sqrt{3}\,x \qquad (\neq \sqrt{3x})$$

$$(-1) \cdot x = -x$$

$$1 \cdot x = x$$

$$0 \cdot x = 0$$

Addition

$$x + x = 2x$$

$x + y$ no simplification

$$x + y + x = 2x + y$$

Multiplication $x \cdot y = xy = yx$

$$x \cdot x = x^2$$

$$xy \cdot x^2 = x \cdot y \cdot x \cdot x = x^3 y$$

Division $\underbrace{\frac{x+4y}{z}}_{\text{rational expression}}$, $\underbrace{\frac{2xy+\sqrt{3}}{3x+y^2}}_{\text{rat'l exp.}}$ ← polynomial ← polynomial

- fractions of polynomials

Other expressions : $\sqrt{x^2+1}$, $\frac{3\sqrt{x}+1}{\sqrt{x}-1}$

We can insert numbers for the variables :

Ex $\frac{2y}{x^2+1}$ with $x=3$, $y=-1$ gives a

number : $\frac{2\cdot(-1)}{3^2+1} = \frac{-2}{9+1} = \frac{-2}{10} = \frac{-1}{5} = -0.2$

- but $\frac{2y}{x^2+1}$ cannot be simplified.

Problem We have a rational expression $\frac{x^2-x-6}{x-3}$

a) Fill in

x	1	5	-2	2	8	3
$\frac{x^2-x-6}{x-3}$	3	7	0	4	10	" $\frac{0}{0}$ " ↑ undefined

b) Find the pattern. Add two to the x-value (except $x=3$)

Shorter (w. alg.) $x+2$ $(x \neq 3)$

Quadratic expansion

$$(x+r)^2 = (x+r)\cdot(x+r) = x\cdot x + x\cdot r + r\cdot x + r\cdot r$$
$$= x^2 + 2rx + r^2$$

Ex $(x+5)^2 = x^2 + 10x + 25$

Ex $13^2 = (10+3)^2 = 10^2 + 2\cdot 3\cdot 10 + 3^2$
$$= 100 + 60 + 9 = 169$$

11.05

Conjungate expansion

$$(x-r)(x+r) = x\cdot x + \cancel{x\cdot r} - \cancel{r\cdot x} - r\cdot r$$
$$= x^2 - r^2$$

Ex $(x-5)(x+5) = x^2 - 25$

Ex $8\cdot 12 = (10-2)\cdot(10+2) = 100-4 = 96$

3. Roots

Ex (definition) The square root of 5 is the positive number a such that $a\cdot a = 5$

(a is in the calculator : $a = 2.2361.....$

We write a as $\sqrt{5}$

Note Negative numbers don't have square roots.

Ex $\sqrt{0} = 0$

<u>Problem</u> Compute (without calculator)

a) $(\sqrt{2}+3)^2 = (\sqrt{2})^2 + 2\cdot 3\cdot\sqrt{2} + 3^2 = \underline{\underline{11+6\sqrt{2}}}$

b) $(\sqrt{5}-1)(\sqrt{5}+1) = (\sqrt{5})^2 - 1^2 = 5-1 = \underline{\underline{4}}$

<u>Ex</u> There are other roots.

$\sqrt[3]{5}$ is the number a such that $a\cdot a\cdot a = 5$

$\sqrt[5]{32} = 2$ since $\underbrace{2\cdot 2\cdot 2\cdot 2\cdot 2}_{\text{5 times}} = 32$

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4. Powers – repeated multiplication

<u>Ex</u> $3\cdot 3\cdot 3\cdot 3 = 3^4$ "three to the power of four"

$4\cdot 4\cdot 4 = 4^3 \quad (\neq 4\cdot 3)$

the base — exponent

<u>Ex</u> $10^2\cdot 10^3 = 10\cdot 10\cdot 10\cdot 10\cdot 10 = 10^5$

$= 10^{2+3}$

so $\boxed{a^n\cdot a^m = a^{n+m}}$

<u>Ex</u> $\dfrac{3^6}{3^4} = \dfrac{3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3}{3\cdot 3\cdot 3\cdot 3} = \dfrac{3\cdot 3\cdot 1}{1} = 3^2 = 3^{6-4}$

(so $3^{-4} = \dfrac{1}{3^4}$)

$$1 = \frac{5^3}{5^3} = 5^{3-3} = 5^0$$

$$\boxed{\frac{a^n}{a^m} = a^{n-m}}$$

Ex $(3^2)^4 = 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 = 3^8 = 3^{2\cdot 4}$

$$\boxed{(a^n)^m = a^{n\cdot m}}$$

5. Order of operations

Problem. Compute

a) $2 + 3 \cdot 4 = \begin{cases} 5 \cdot 4 = 20 = (2+3)\cdot 4 \\ 2+12 = \underline{14} = 2 + (3\cdot 4) \end{cases}$

b) $2 \cdot 2^4 = \begin{cases} (2\cdot 2)^4 = 256 \\ 2 \cdot (2^4) = \underline{32} \end{cases}$

6. Absolute value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Ex $|3| = 3$, $|-3| = 3$