EBA 1180, lecture 11, 1 Oct. 2024, Runar Ile

Plan
1. Second degree polynomial functions with problem 5a-e, 7a and 8b.
2. Revenue, cost and profit with problem 9.

1. Second degree polynomial functions.

5a) Because we have two easy-to-read zeros, we use the form $f(x) = a(x-r_1)(x-r_2) = a(x-2)(x-5)$

roots

To find a, note that

$f(0)=5$ so $a\cdot(0-2)\cdot(0-5) = 5$ (equation for a)

$$a \cdot 10 = 5$$

$$a = \frac{5}{10} = \frac{1}{2} = 0.5$$

and $\underline{f(x) = \frac{1}{2}(x-2)(x-5)}$

5b) $x=2$ is the larger root,

$f(-1) = 6 = f(0)$, so $x = -\frac{1}{2}$ is the symmetry line (in the middle between -1 and 0)

So the smaller root has to be

$x = -\frac{1}{2} - 2.5 = -3$

Hence $f(x) = a(x-2)(x+3)$

To find a, note $f(0) = 6$, so

$$a\cdot(0-2)\cdot(0+3) = 6$$

$$a\cdot(-6) = 6$$

$$a = \frac{6}{-6} = -1$$

and $\underline{f(x) = -(x-2)(x+3)}$

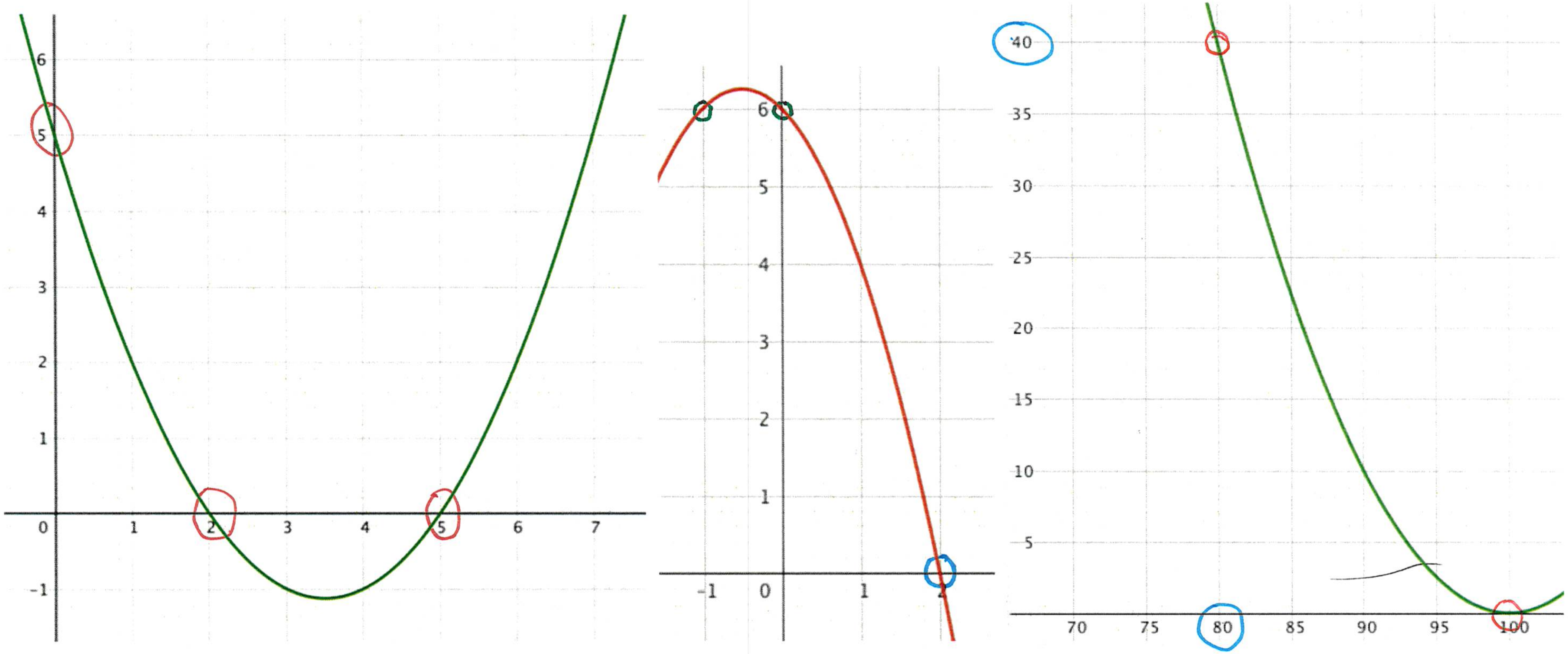


Figure 2: Parabolas (a-c)

5c) We see that $x = 100$ is a double root

$$f(x) = a(x-100)(x-100) = a(x-100)^2$$

Since $(80, 40)$ is a point on the graph,

$f(80) = 40$, so $a \cdot (80-100)^2 = 40$

$$a \cdot (-20)^2 = 40$$

and so

$$a \cdot 400 = 40$$

$$\underline{\underline{f(x) = \frac{1}{10}(x-100)^2}} \qquad a = \frac{40}{400} = \frac{1}{10} = 0.1$$

This is the std. form $f(x) = a(x-s)^2 + d$

with $a = \frac{1}{10}$, $s = 100$ and $d = 0$.

5d) We observe the symmetry axis $x = 1$

and the maximum value $y = -1$.

Then $f(x) = a(x-s)^2 + d$

$$= a(x-1)^2 - 1$$

To find a, note $f(0) = -2$, we get

$$a \cdot (0-1)^2 - 1 \stackrel{eq.}{=} -2$$

$$a - 1 = -2$$

$$a = -2 + 1 = -1$$

and $\underline{\underline{f(x) = -(x-1)^2 - 1}}$

5e) The symmetry axis is $x = -3$

The minimum value is $y = 4.25$

(in the middle between 4 and 4.5)

so $f(x) = a(x+3)^2 + 4.25$

From $f(-2) = 4.5$ we get

$$a \cdot (-2+3)^2 + 4.25 \overset{eq.}{=} 4.5$$

$$a + 4.25 = 4.5$$

$$a = 4.5 - 4.25 = 0.25$$

and $\underline{\underline{f(x) = 0.25(x+3)^2 + 4.25}}$

7a) Three points on the graph: $P = (0, 7)$

$Q = (1, 4)$

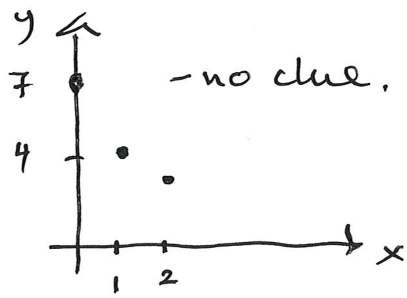
$R = (2, 3)$

only one good info:

Graph crosses y-axis at 7.

So use the form

$f(x) = ax^2 + bx + c$



P: $f(0) = 7$ gives $\underline{c = 7}$

Q: $f(1) = a \cdot 1^2 + b \cdot 1 + 7 = 4$

$\boxed{a + b = -3}$ (1)

R: $f(2) = a \cdot 2^2 + b \cdot 2 + 7 = 3$

$\boxed{4a + 2b = -4}$ (2)

Solve this system of equations. Multiply (1) on each side by 4 and get

$$4a + 4b = -12$$

subtract (2):

$$4a + 2b = -4$$

$$0a + 2b = -8$$

so $b = \frac{-8}{2} = \underline{-4}$

From (1) we get $a = -3-(-4) = \underline{1}$

so $\underline{\underline{f(x) = x^2 - 4x + 7}}$

8b) $f(x) = 3x^2 + 36x + 110$

How to complete the square?

Note tha $3x^2 + 36x = 3 \cdot [x^2 + 12x]$

complete the square of $x^2 + 12x$:

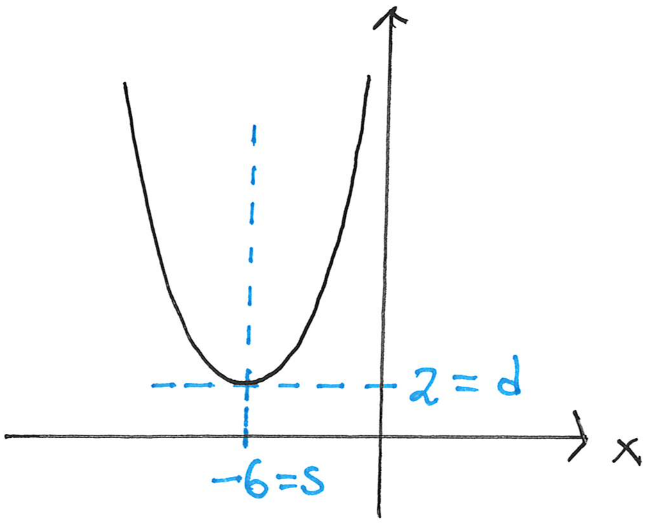
$x^2 + 12x = (x+6)^2 - 36$

So $f(x) = 3[(x+6)^2 - 36] + 110$

$= 3(x+6)^2 - 108 + 110$

$= \underline{\underline{3(x+6)^2 + 2}}$

so $\underline{\underline{a = 3}}$, $\underline{\underline{s = -6}}$, $\underline{\underline{d = 2}}$



Start : 11.05

Summary: second degree polynomial functions

3 standard forms:

A) If we know the roots: $f(x) = a(x-r_1)(x-r_2)$

B) If we know the symmetry axis: $x = s$

and the max/min value: $y = d$

then $f(x) = a(x-s)^2 + d$

C) Other cases: $f(x) = ax^2 + bx + c$

(but we can always use B).

2. Revenue, cost and profit

x = number of units produced and sold

p = unit price, so revenue $R(x) = p \cdot x$

Probl 9. Determine p such that the profit is positive (exactly) when $x > 300$.

a) The cost function is $C(x) = 2100 + 5x$.

The profit function is $P(x) = R(x) - C(x)$

$= px - (2100 + 5x) = (p-5)x - 2100$

By assumption the inequality $P(x) > 0$ should have the solutions $x > 300$

We solve the inequality

$P(x) > 0$, that is

$(p-5)x - 2100 > 0 \quad | +2100 \quad (p-5)x > 2100 \quad | : (p-5)$

Assume

$\underline{p-5>0}$: $\quad x > \dfrac{2100}{p-5}$ then

$$\frac{2100}{p-5} = 300 \quad \text{(same solutions)}$$

- we solve this eq: for p

$$2100 = 300(p-5)$$

$$2100 = 300p - 1500$$

$$300p = 3600$$

$$p = \frac{3600}{300} = \underline{12}$$

(and then $p-5 = 7 > 0$ - ok!)

Assume

$\underline{p-5<0}$: $\quad x < \dfrac{2100}{p-5}$ - is a negative number.

But the number of units produced is positive
- so no solutions in this case.

9b) The cost function is $C(x) = 4500 - 5x + 0.01x^2$
with $x \in [0, 1000]$.

Then $P(x) = px - (4500 - 5x + 0.01x^2)$

resolve and collect terms

$$= -0.01x^2 + (p+5)x - 4500$$

complete the square

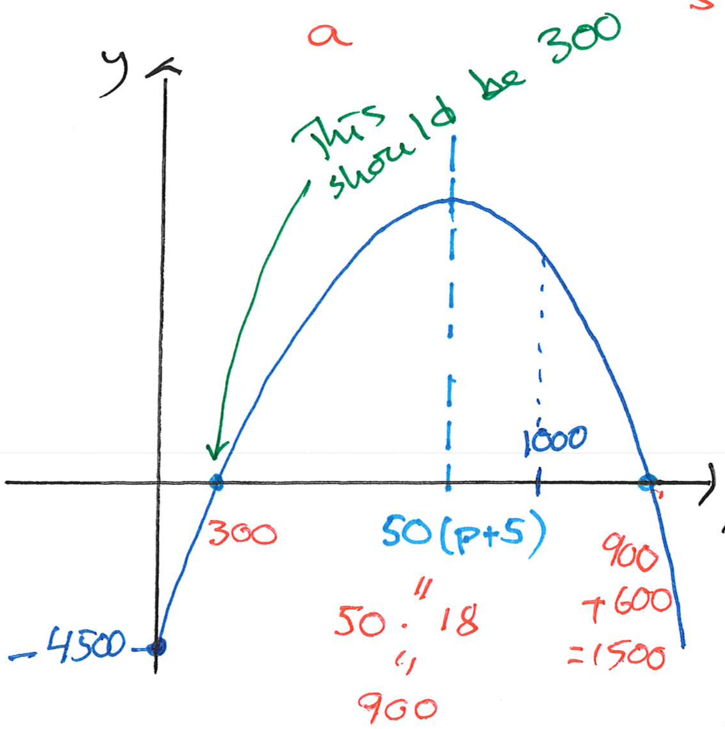
$$= -0.01\left(x^2 - \underset{\downarrow :2}{100}(p+5)x\right) - 4500$$

$$= -0.01\left(\left[x - 50(p+5)\right]^2 - 50^2(p+5)^2\right) - 4500$$

square

-multiply -0.01 back inside the parenth.

$$= \underbrace{-0.01}_{a}\left[x - \underbrace{50(p+5)}_{s}\right]^2 + \underbrace{25(p+5)^2 - 4500}_{d}$$



Need to find the value of p which makes the smaller root of P(x) equal to 300.

solve eq. $P(300) = 0$ for p.

that is $-0.01 \cdot 300^2 + (p+5)\cdot 300 - 4500 = 0$

and solve:

$$(p+5)\cdot 300 = 4500 + 900$$

$$\text{so } p+5 = \frac{5400}{300} = 18$$

$$\text{so } \underline{\underline{p = 13}}$$

Note Positive profit for $x \in \langle 300, 1000]$

– look at the graph!