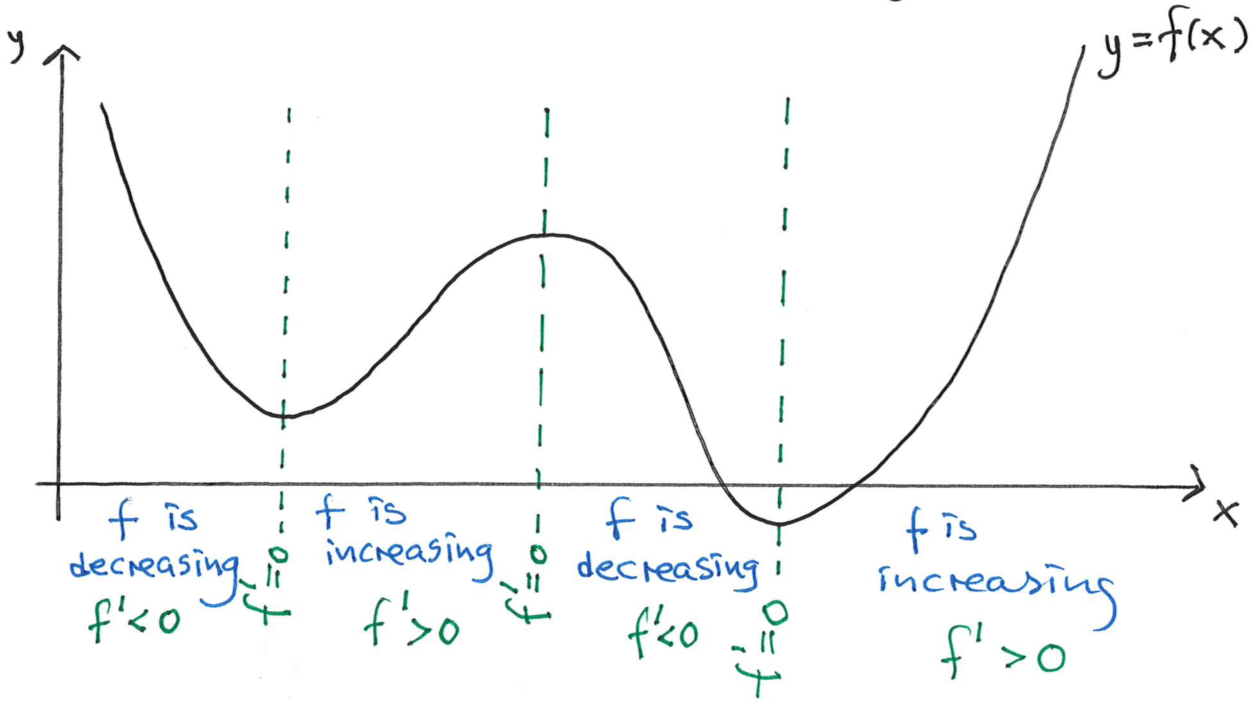
EBA 1180, lecture 18, 23 Oct. 2024, Runar Ile

Plan: 1. Local max/min and stationary points
2. Global max/min
3. The mean value theorem

1. Local max/min and stationary points



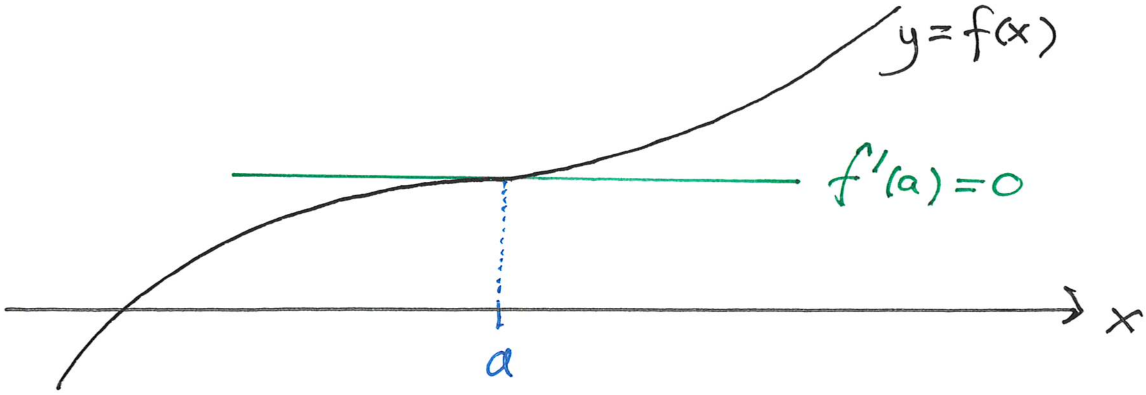
When $f'(x)$ is positive, $f(x)$ is increasing
When $f'(x)$ is negative, $f(x)$ is decreasing

Important conclusion The sign diagram of $f'(x)$ determines where $f(x)$ is increasing and decreasing.

If $x=a$ is a local minimum point, then $f'(a)=0$ and $f'(x)$ changes sign from $-$ to $+$

If $x=a$ is a local maximum point, then $f'(a)=0$ and $f'(x)$ changes sign from $+$ to $-$

Ex



Here $x=a$ is neither a loc. max. point nor a loc. min. point.
It is a terrace point.

Definition: If $f'(a)=0$ then $x=a$ is a stationary point.

Ex $f(x) = x^3 - 6x^2 + 5$. We find the stationary points by simply solving the eq. $f'(x)=0$.

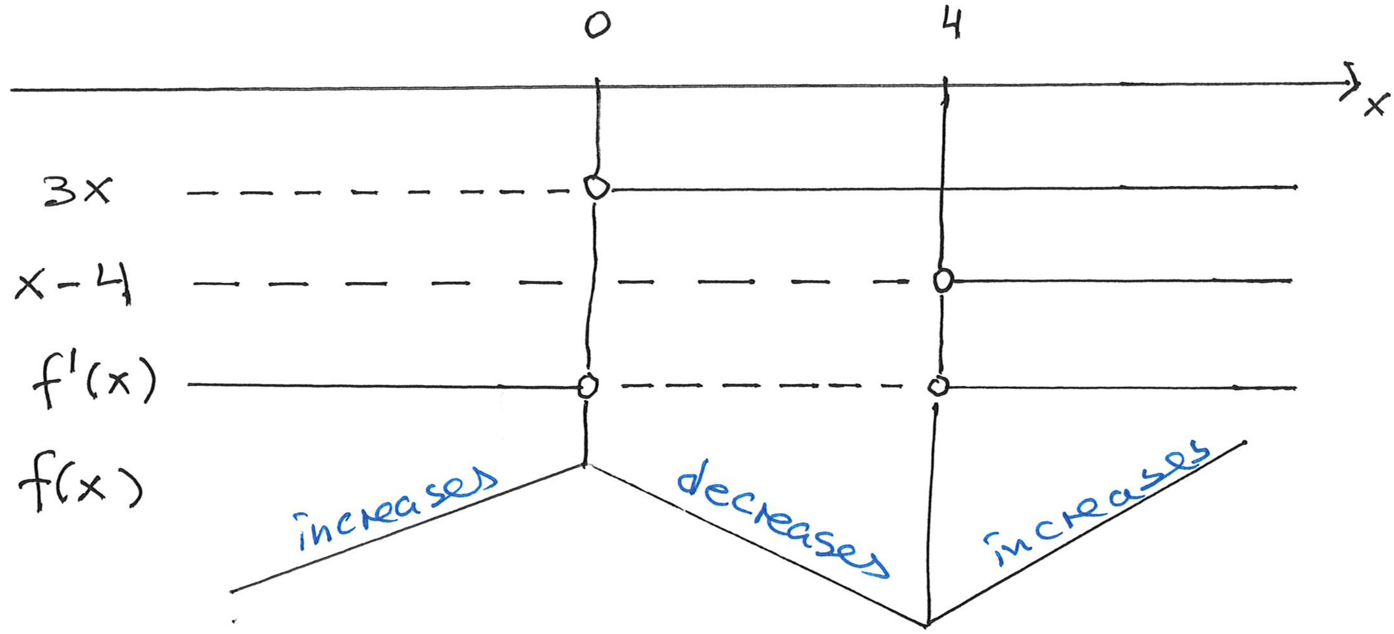
First we find $f'(x) = 3x^2 - 6 \cdot 2x + 0$

$$= 3x^2 - 12x$$

$$= 3x(x-4)$$

So $f'(x)=0$ has solutions $x=0$ and $x=4$

Where is $f(x)$ increasing/decreasing?
- we determine the sign of $f'(x)$ by a sign diagram.



$f(x)$ is strictly increasing for $x \leq 0$ (so $x \in \langle \leftarrow, 0]$)

$f(x)$ is strictly decreasing for $0 \leq x \leq 4$ (so $x \in [0, 4]$)

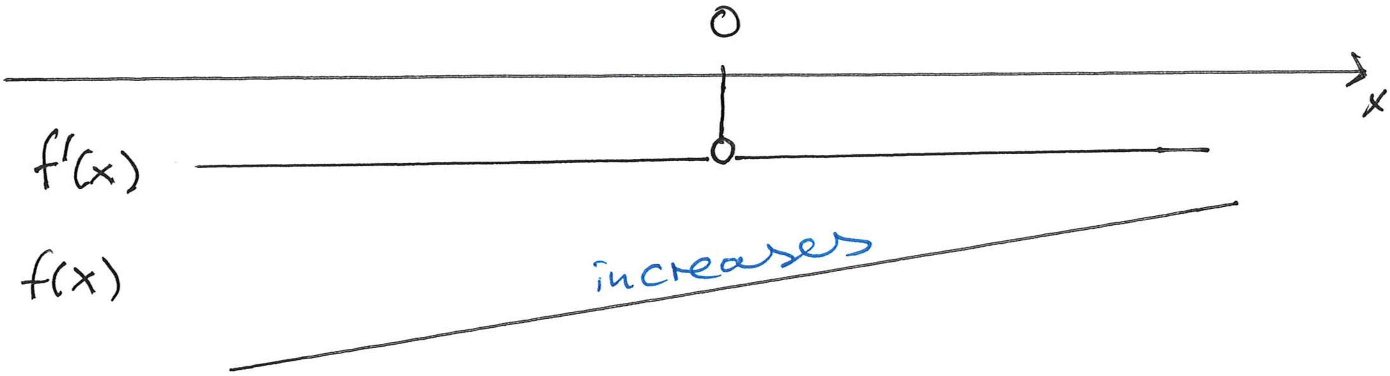
$f(x)$ is strictly increasing for $x \geq 4$ (so $x \in [4, \rightarrow\rangle$)

Then $x=0$ is a local maximum point

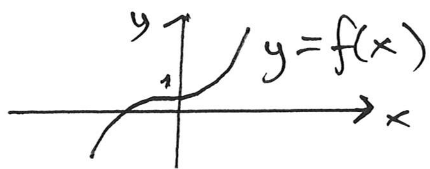
and $x=4$ —— " —— min. —— " ——

<u>Ex</u> $f(x) = x^3 + 1$, so $f'(x) = 3x^2$ and

<u>$x = 0$</u> is the only stationary point for $f(x)$.



<u>Conclusion</u> $f(x)$ is strictly increasing for all numbers on the number line.



$(x \in \mathbb{R})$

Start: 11.01

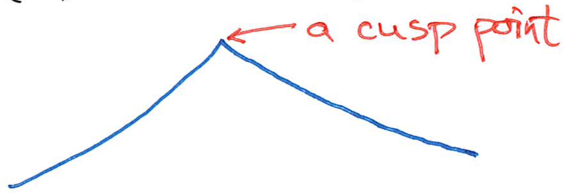
2. Global max/min

> **The extreme value theorem** If $f(x)$ is a continuous function (graph is one snake) on the interval $D_f = [a, b]$ then $f(x)$ has a global maximum and a global minimum.

Possible max/min points:

(*) stationary points (solve $f'(x) = 0$)

(*) Cusp points (where $f'(x)$ is not defined)



(*) End points $(x=a,\ x=b)$

<u>Ex</u> $f(x) = x^3 - 6x^2 + 5$ and $D_f = [-1, 7]$

Find the max./min. of $f(x)$.

<u>Solution</u>

(*) stationary points: $f'(x) = 3x^2 - 12x = 0$

gives $\underline{x=0}$, $\underline{x=4}$

(*) cusp points: none ($f'(x)$ defined everywhere)

(*) end points: $\underline{x=-1}$, $\underline{x=7}$

These four points are my candidate points for max/min.

Calculates:

$f(-1) = -2$ $\quad$ $f(4) = -27$

$f(0) = 5$ $\quad$ $f(7) = 54$

So $x=4$ gives the glob. minimum

$$f(4) = \underline{\underline{-27}}$$

and $x=7$ gives the glob. max.

$$f(7) = \underline{\underline{54}}$$

Ex $f(x) = 12 - x$ with $D_f = [3, 10]$

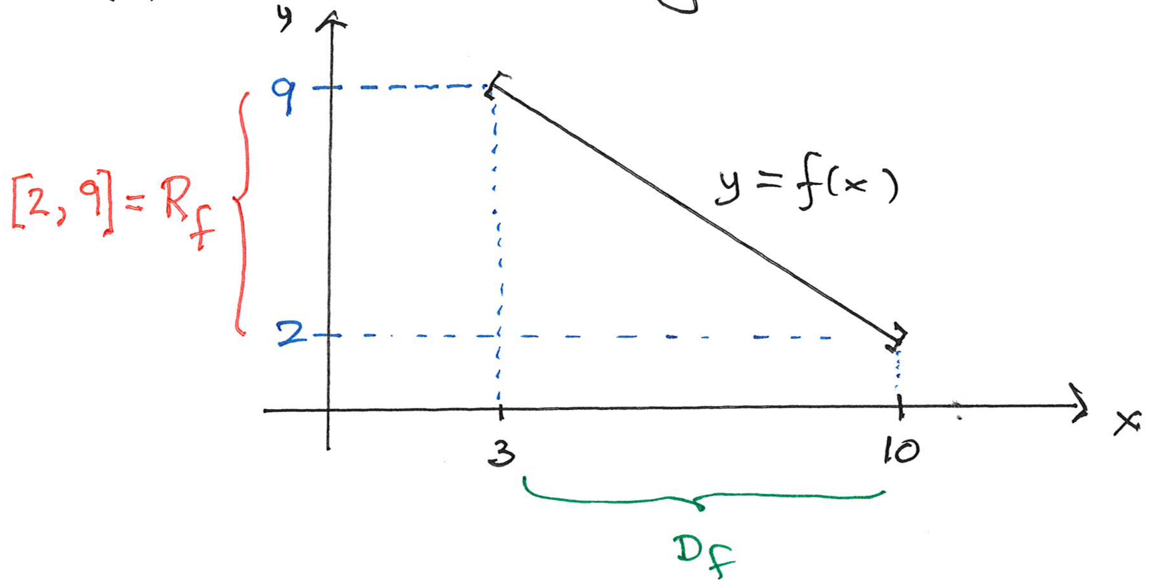
(*) $f'(x) = -1$, never $= 0$: no stationary points

(*) no cusp points ($f'(x)$ is defined everywhere)

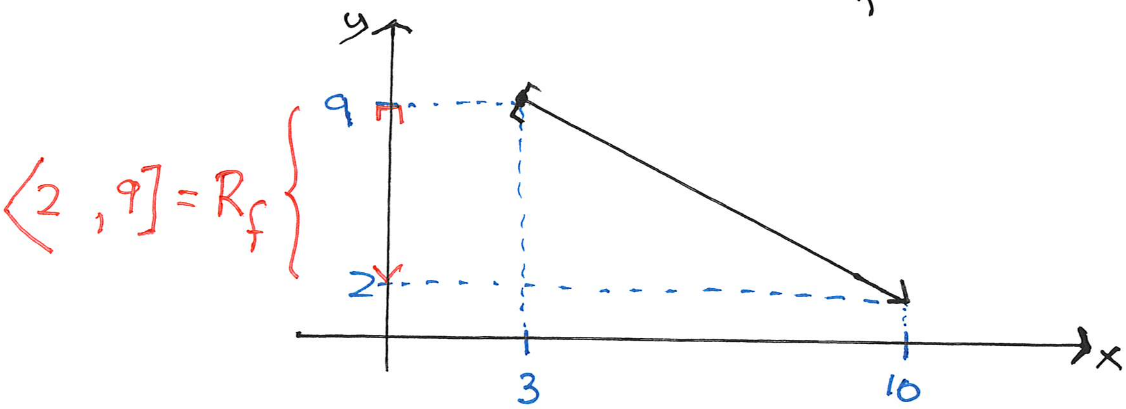
(*) end points : $x = 3$ is a max. point

$x = 10$ is a min. point

($f(x)$ is decreasing in the whole domain)



Ex $f(x) = 12 - x$ with $D_f = [3, 10\rangle$

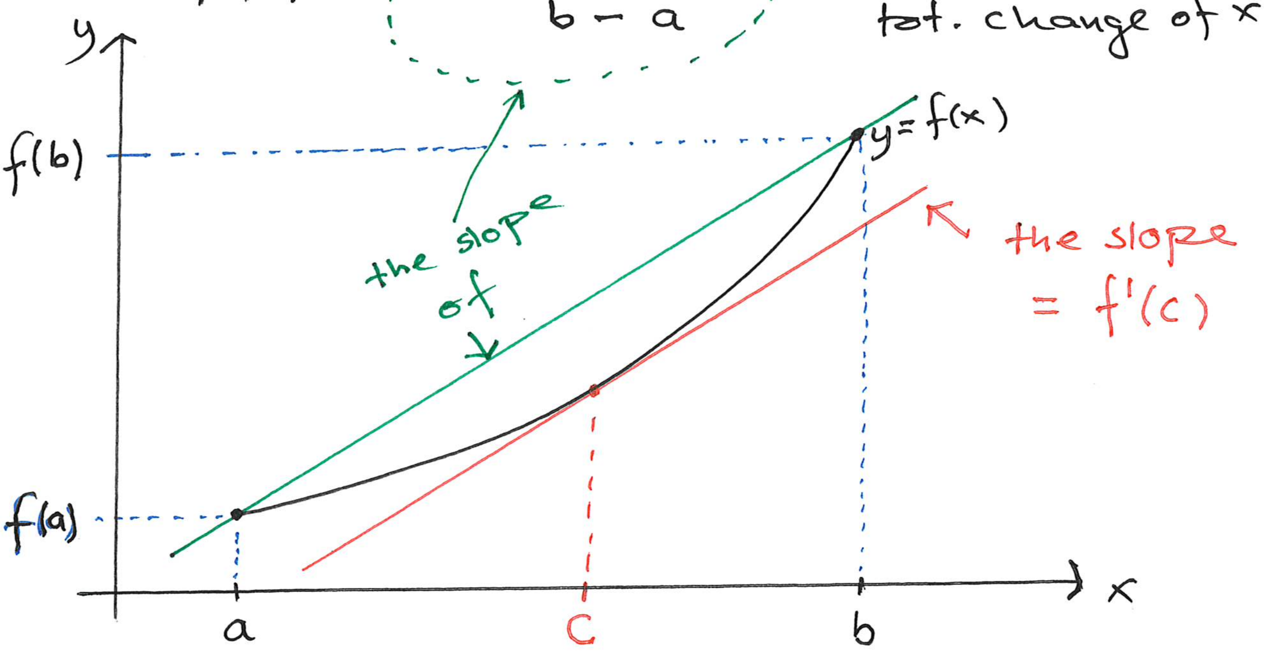


So $x = 3$ is still the max. point with max. value $f(3) = 9$, but there are no min. points and no min. value.

3. The mean value theorem

If $f(x)$ is continuous (connected graph) in the interval $[a, b]$ and differentiable (no cusps) then there is a number c between a and b ($a < c < b$) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{\text{tot. change of } y}{\text{tot. change of } x}$$



green line and red line are parallell (same slope)

Ex $f(x) = e^x + x^2$. Then $f(0) = e^0 + 0^2 = 1$ and $f(1) = e^1 + 1^2 = e + 1$ (so $a = 0$, $b = 1$)

By the mean value theorem there is a number c between 0 and 1 such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{e + 1 - 1}{1} = e$$

Note $f'(x) = e^x + 2x$ (easy), but we cannot find an exact solution to the eq. $e^x + 2x = e$