EBA 1180, lecture 19, 29 Oct. 2024, Runar Ile

Plan

1. Repetition with problems from last week:
   - Probl 1c: draw two graphs
   - Probl 2 b, d, h, i, k
     interpretations of the graph of $f'(x)$.
   - Probl 3c: which graph is $f(x) / f'(x)$?
   - Probl 4g: increasing / decreasing from $f'(x)$.

2. Implicit differentiation

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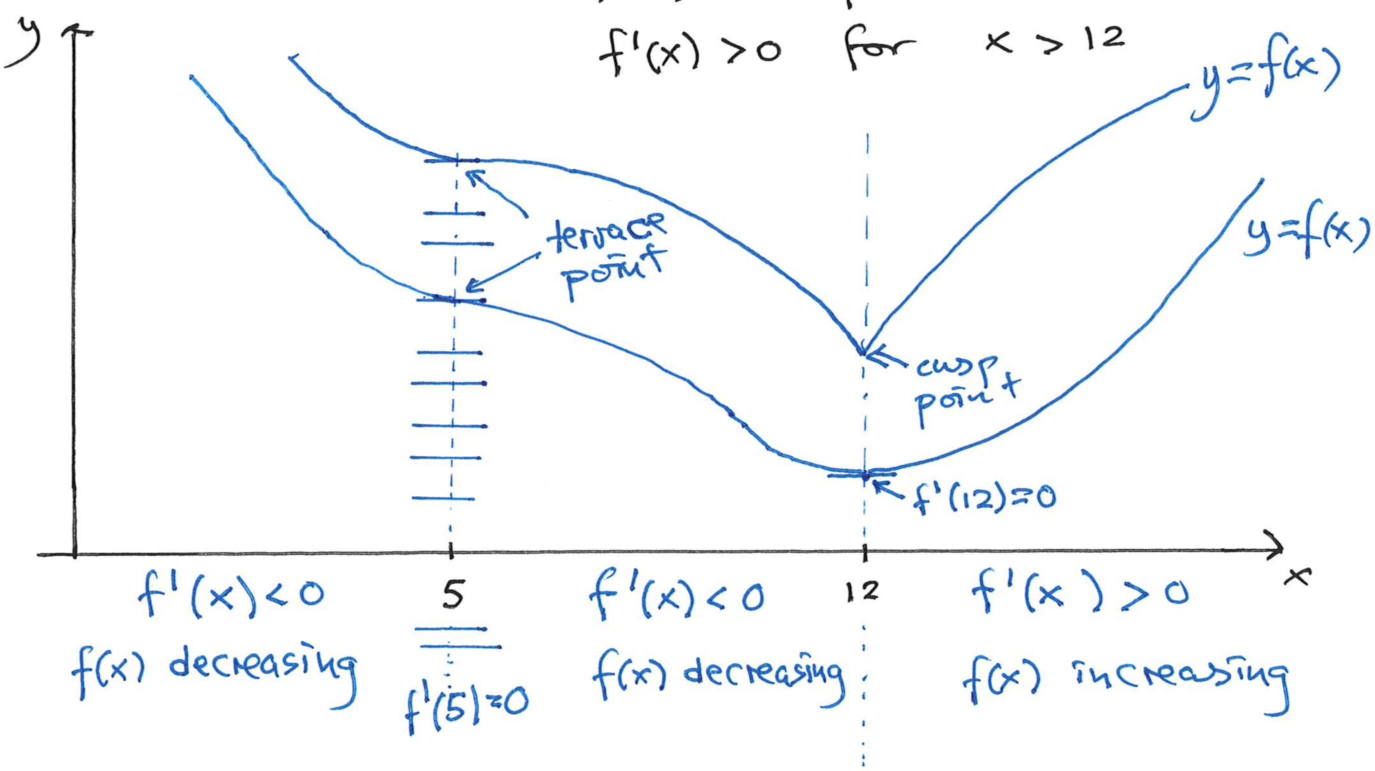
1. Repetition Probl 1c

$f'(x) < 0$ for $x < 5$, $f'(5) = 0$

$f'(x) < 0$ for $5 < x < 12$

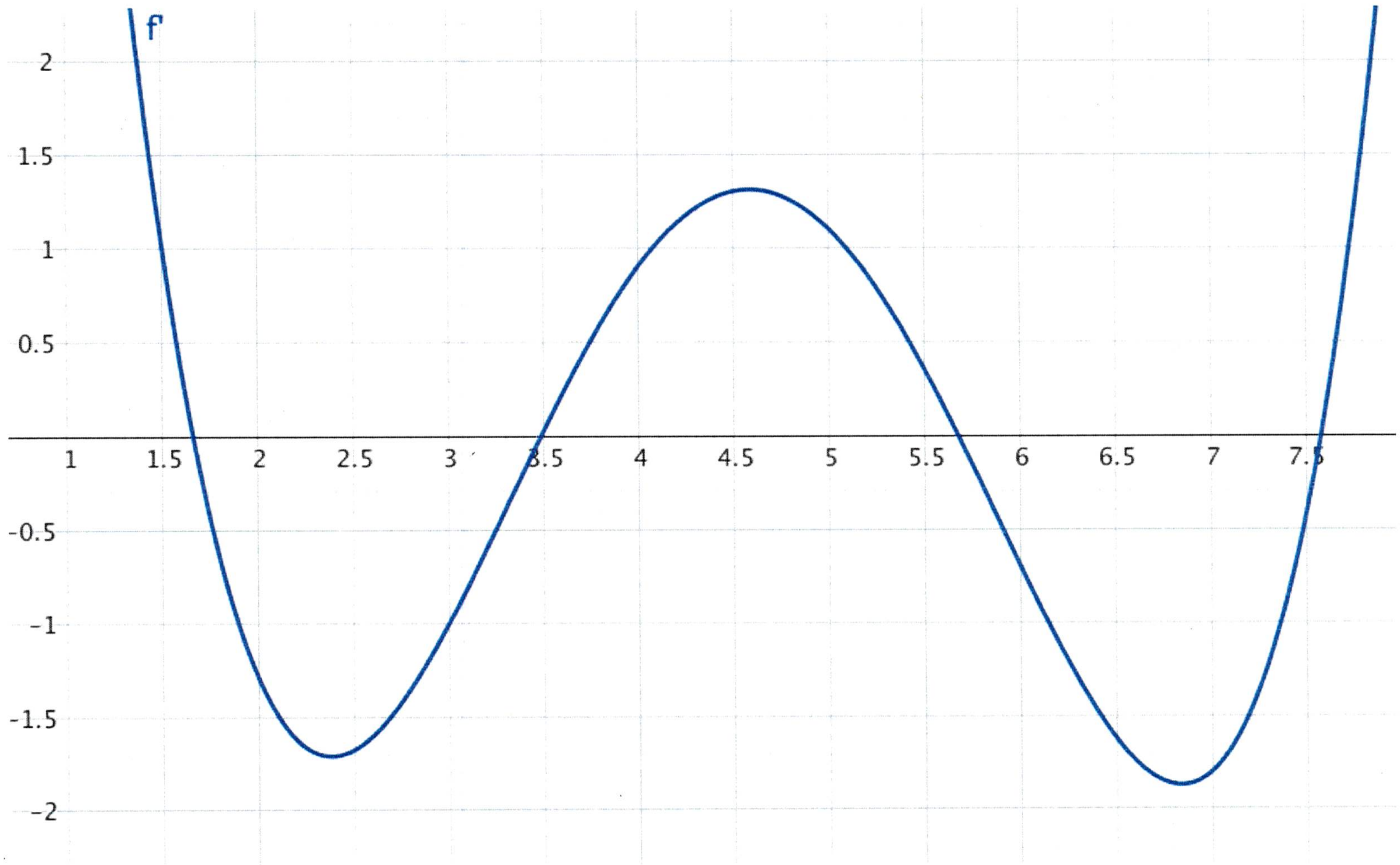
$f'(x) > 0$ for $x > 12$



Probl 2 b) $f(2) < f(3)$ FALSE.

We see (from the graph of $f'(x)$) that $f'(x) < 0$ for $x \in [2, 3]$. Hence $f(x)$ is strictly decreasing for $x \in [2, 3]$ and $f(2) > f(3)$.

Oppgave 2 I figur 1 ser du grafen til $f'(x)$.



Figur 1: Grafen til $f'(x)$

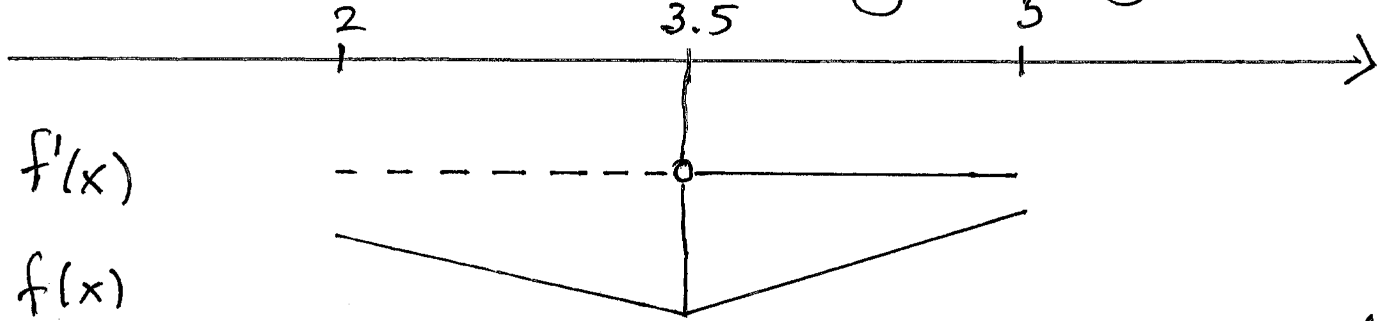
2d) $f(x)$ has a (local) minimum at $x = 3.5$.

TRUE.

We have $f'(x) < 0$ for $x \in [2, 3.5\rangle$

and $f'(x) > 0$ for $x \in \langle 3.5, 5]$

and $f'(3.5) = 0$. Sign diagram:



Conclusion: $x = 3.5$ is a loc. min. point for $f(x)$.

2h) $f(x)$ increases faster around $x = 1.5$ than around $x = 5.5$. TRUE

The slope of the tangent of $f(x)$ at $x = 1.5$ is approx. 1 (since $f'(1.5) \approx 1$)

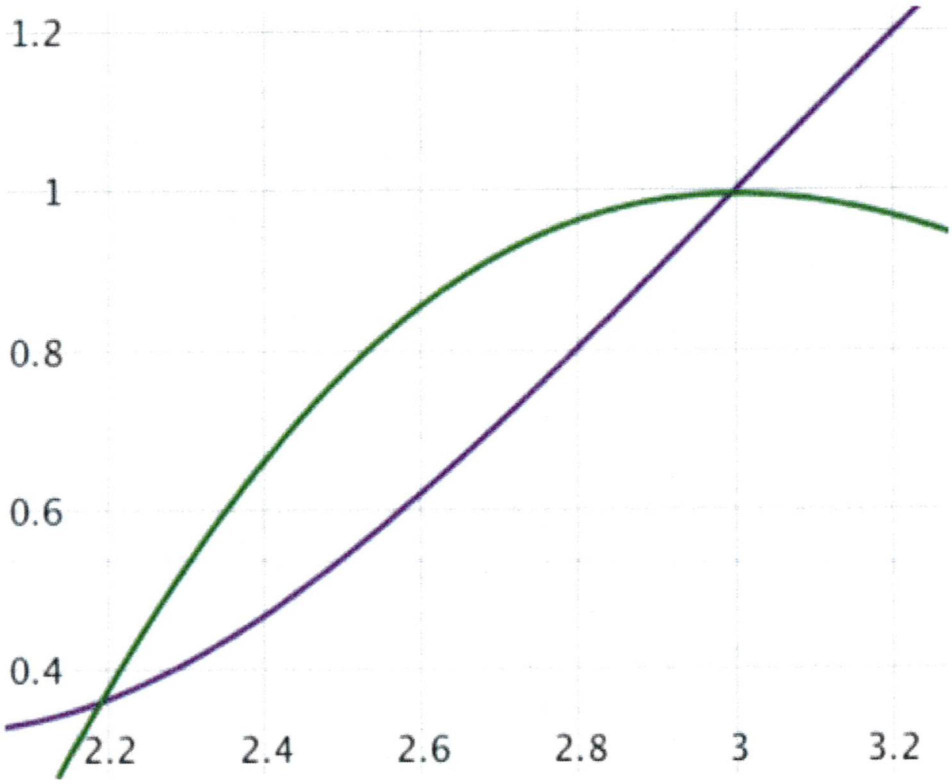
The slope of the tangent of $f(x)$ at $x = 5.5$ is approx. 0.35 (since $f'(5.5) \approx 0.35$).

2i) The derivative of $f'(x)$ is positive for $x = 7.6$.

TRUE because the slope of the tangent of $f'(x)$ is (very) positive for $x = 7.6$ (maybe $f''(7.6) \approx 6$)

2k) We cannot use the graph of $f'(x)$ to determine if $f(4.5)$ is positive.

TRUE: If we add or subtract 1 mill. to $f(x)$ $f'(x)$ is not changed.


1.2
1
0.8
0.6
0.4
2.2
2.4
2.6
2.8
3
3.2

Probl 3c Which graph is $f(x)$ / $f'(x)$?

I guess $f(x)$ is the violet one. But much easier to determine what is wrong!

Assume $f(x)$ is the green. Then $f'(x)$ is violet.

But the slope of the green is negative for $x > 3$ while the values of the violet are bigger than 1. So the assumption is wrong. The only ~~other~~ possibility is that $f(x)$ is the violet one and $f'(x)$ is the green.

Start: 11.02

Probl 4g $f'(x) = e^{2x} - 4x + 3$.

Where is $f(x)$ strictly increasing/decreasing?

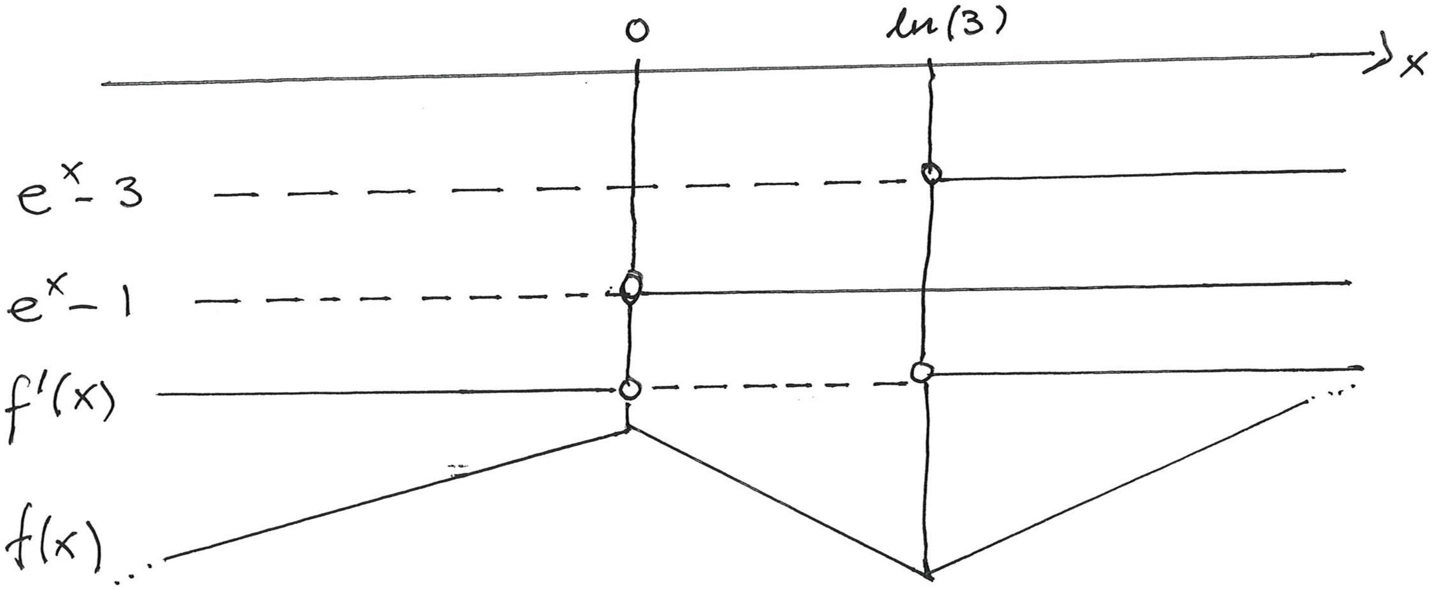
We want to use the sign diag. of $f'(x)$.

But we have to factorise $f'(x)$ first.

Put $u = e^x$. Then $u^2 = e^x \cdot e^x = e^{2x}$

so $f'(x) = u^2 - 4u + 3 = (u-3)(u-1)$

$f'(x) = (e^x - 3)(e^x - 1)$. Sign diag:



So $f(x)$ is strictly increasing for x in $\langle \leftarrow, 0]$

—— " —— decreasing —— " —— $[0, \ln(3)]$

—— " —— increasing —— " —— $[\ln(3), \rightarrow\rangle$

Stationary points for $f(x)$:
- the solutions of the equation $f'(x) = 0$ are stationary points.

Here $\underline{\underline{x = 0}}$ and $\underline{\underline{x = \ln(3)}}$

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2. Implicit differentiation

Ex $\quad f(x) = \frac{1}{x} = x^{-1}$

$$f'(x) = (-1) \cdot x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$$

- usual differentiation.

Instead put $y = f(x)$, so $y = \frac{1}{x} \quad | \cdot x$

and get $\boxed{xy = 1}$ - eq. with both x and y.

Differentiate each side of the eq.
with respect to x and
think about y as a function on x, so $y = y(x)$

$$(x \cdot y)'_x = (1)'_x$$

the product rule on the LHS gives

$$(x)'_x \cdot y + x \cdot (y)'_x = 0$$

$$1 \cdot y + x \cdot y'_x = 0$$

We can solve this equation for y'_x :

$$x \cdot y' = -y \quad | : x$$

$$\boxed{y' = -\frac{y}{x}}$$

(Note : $y = \frac{1}{x}$ so $y' = -\frac{\left(\frac{1}{x}\right)}{x} = -\frac{1}{x^2}$)

This is called implicit differentiation.

<u>One application</u> Can use this to find slopes of tangents to the curve defined by the original equation $(xy = 1)$.

E.g. if $\underline{x = 2}$ then $xy = 1$ gives $2y = 1 \quad | : 2$

so $\underline{y = \frac{1}{2}}$

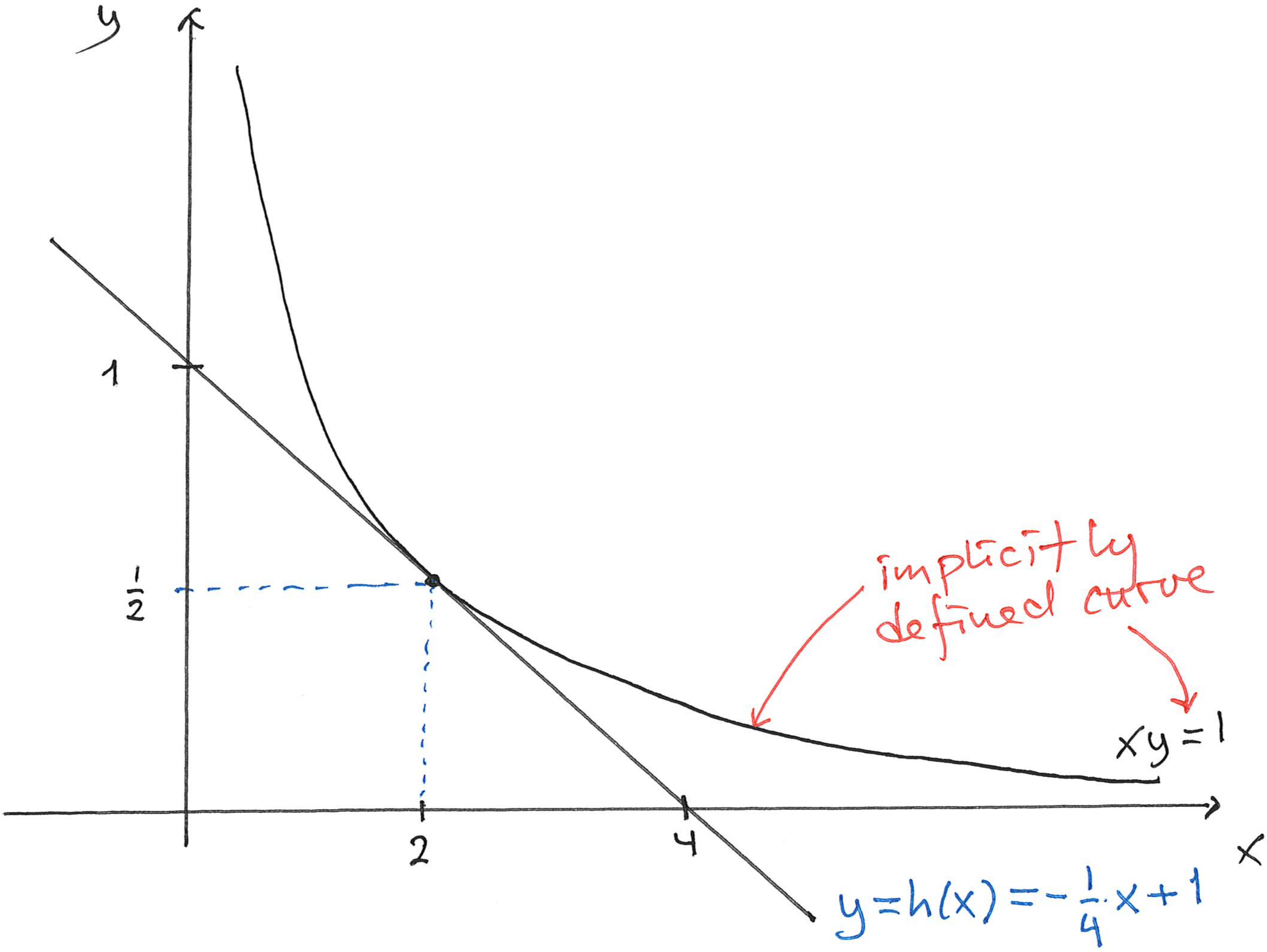
Also $y'\Big|_{\substack{x=2 \\ y=\frac{1}{2}}} \overset{\text{red box}}{=} -\frac{\left(\frac{1}{2}\right)}{2} = -\frac{1}{4}$

Can apply this to find the equation expression $h(x)$ of the tangent of the curve $xy = 1$ at the point $\left(2, \frac{1}{2}\right)$ by the point-slope formula

$$h(x) - \frac{1}{2} = -\frac{1}{4} \cdot (x - 2)$$

the slope

so $\underline{\underline{h(x) = -\frac{1}{4} \cdot x + 1}}$


y
x
1
$\frac{1}{2}$
2
4
implicitly defined curve
$xy=1$
$y=h(x)=-\frac{1}{4}\cdot x+1$